Composite Functions
For two functions $f$ and $g$, the composition of $f$ with $g$ evaluated at $x$ is $f(g(x))$.

• Think of composition of functions in this way: just as we substitute a number into a function, we may substitute a function into a function.

• The domain of the function $f \circ g$ ($f$ composed with $g$) is the set of all numbers $x$ in the domain of $g$ such that $g(x)$ is in the domain of $f$.

-Examples:

(a)

(b)

(c)

Note from the examples that $f(g(x))$ is not necessarily the same as $g(f(x))$!

-Example: An insurance company keeps reserves (money to pay claims) of $R(v) = 2v^{0.3}$, where $v$ is the value of all its policies, and the value of its policies is predicted to be $v(t) = 60 + 3t$, where $t$ is the number of years from now. (Both $R$ and $p$ are in millions of dollars.) Express the reserves $R$ as a function of $t$, and evaluate the function at $t = 10$. 
Shifts of Graphs
By applying certain transformations to the graph of a given function we can obtain the graphs of certain related functions. This will give us the ability to sketch the graphs of many functions quickly by hand. It will also enable us to write equations for given graphs.

In general, adding to or subtracting from the $x$-value means a horizontal shift, while adding to or subtracting from the function means a vertical shift.

Suppose $c > 0$. To obtain the graph of:
- $y = f(x) + c$, shift the graph of $y = f(x)$ a distance $c$ units upward
- $y = f(x) - c$, shift the graph of $y = f(x)$ a distance $c$ units downward
- $y = f(x - c)$, shift the graph of $y = f(x)$ a distance $c$ units to the right
- $y = f(x + c)$, shift the graph of $y = f(x)$ a distance $c$ units to the left

-Examples:

(a)

(b)

Difference Quotients
Example:

Definition: For a function $f$, the quantity $\frac{f(x + h) - f(x)}{h}$ is called the difference quotient.
Examples: Given the function $f$, find and simplify $\frac{f(x + h) - f(x)}{h}$.

(a)

(b)

Section 1.5 - Exponential Functions

Definition: A function that has a variable in an exponent is called an exponential function.

-Examples:

(a)

Base:

Table of Values:

<table>
<thead>
<tr>
<th>$x$</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph:
We can define an exponential function $f(x) = a^x$ for any positive base $a$. We always take the base to be positive.

- Exponential functions with bases $a > 1$ are used to model growth, as in populations or savings accounts.
- Exponential functions with bases $a < 1$ are used to model decay, as in depreciation.
- What happens if $a = 1$?