1. Which of the numbers below is equal to $\sqrt[4]{4}\sqrt[4]{8}$?

(a) $\sqrt{12}$  (b) $\sqrt[12]{32}$  (c) $2\sqrt[12]{32}$  (d) $\sqrt[3]{32}$  (e) none of these

2. Let $A$, $B$ and $C$ be the points $(1, 3)$, $(5, -1)$ and $(3, 3)$ respectively. If $\triangle MNP$ is the triangle whose vertices are the midpoints of the sides of $\triangle ABC$, then the shortest side of $\triangle MNP$ measures:

(a) $\sqrt{2}$  (b) $\sqrt{2}/2$  (c) 1  (d) $\sqrt{5}$  (e) none of these

3. At a certain time there are $n$ people at a party. After that, 31 women leave the party, leaving twice as many men as women in the party. Later, 55 men leave, leaving three times as many women as men in the party. What is the number $n$?

(a) 100  (b) 115  (c) 105  (d) 130  (e) none of these

4. What number below is equal to $\log_3\left(\frac{27}{\sqrt[3]{9}}\right)$?

(a) $\frac{7}{3}$  (b) $-\frac{1}{3}$  (c) $\frac{1}{3}$  (d) $\frac{1}{2}$  (e) none of these
5. In the picture below, $ABCD$ is a square and $ABE$ is an equilateral triangle.

The measure of $\angle AED$ in degrees is:

(a) 20   (b) 5   (c) 10   (d) 15   (e) none of these

6. Let $x_1$, $x_2$ and $x_3$ be three real numbers such that $x_1 - x_2$ and $x_1 + x_2 + x_3$ are both rational. Consider the following statements:

(I) If either $x_1$ or $x_2$ is rational, then $x_3$ is rational.

(II) If $x_3$ is rational, then so is $x_1 + x_2$.

(III) If $x_3$ is rational, then so are both $x_1$ and $x_2$.

Then, the complete list of necessarily true statements is:

(a) I   (b) I and II   (c) II   (d) I, II and III   (e) none of these

7. The segment $AB$ is the side of a regular hexagon of area $\sqrt{3}$. A point $P$ in the perpendicular bisector of the segment $AB$ makes a triangle $ABP$ of area $\sqrt{2}$. Then, the distance from $P$ to $AB$ is:

(a) $2\sqrt{3}$   (b) $3\sqrt{2}$   (c) $\sqrt{2}$   (d) $\sqrt{3}$   (e) none of these
8. If four times the reciprocal of the circumference of a circle equals the diameter of
this circle, then its area is:

(a) 1  (b) 1/π  (c) 1/π²  (d) π²  (e) none of these

9. Three numbers from the first 99 positive integers are chosen at random (with
repetitions allowed). What is the probability that the sum is divisible by 3?

(a) 1/3  (b) 5/27  (c) 32/99  (d) 1/5  (e) none of these

10. A circle in the Cartesian plane is tangent to the x-axis at the point (5, 0) and
passes through the point (1, 2). What is the radius of this circle?

(a) √5  (b) 3√5  (c) 2√5  (d) 5  (e) none of these

11. The real number $x$ is such that $0 < x < \pi$ and

$$\log_3(1 - \cos(x)) + \log_3(1 + \cos(x)) = -2.$$ 

Then, $\cos(2x) + \sin(x)$ is:

(a) 10/9  (b) 1/3  (c) 7/9  (d) 8/9  (e) none of these

12. Suppose $p(x)$ is a polynomial with $p(1) = -1$ and $p(-1) = 1$. The remainder
upon dividing $p(x)$ by $x^2 - 1$ is:

(a) $-x$  (b) $x - 1$  (c) $-x + 1$  (d) $x$  (e) none of these
13. In the figure below we have that $AE$ is parallel to $CD$, $BC$ is parallel to $DE$, $AE$ measures 2, $\angle ABC$ measures 45 degrees and $\angle BCD$ measures 75 degrees.

![Diagram of geometric figures]

The distance from the point $E$ to the segment $AB$ is:

(a) $\sqrt{3}$  (b) $\sqrt{3}/2$  (c) $\sqrt{2}$  (d) 1  (e) none of these

14. Let $a = 2^6 \cdot 3^4 \cdot 7^5$. Then, the number of divisors of $a$ which are relatively prime to $15^{2012}$ and for which the least common multiple with 14 is divisible by 28 is:

(a) 25  (b) 125  (c) 30  (d) 42  (e) none of these

15. Three fair dice are thrown. What is the probability that the three numbers on the top sides are (possibly after rearrangement) consecutive?

(a) $\frac{2}{9}$  (b) $\frac{5}{9}$  (c) $\frac{1}{9}$  (d) $\frac{1}{4}$  (e) none of these

16. Let $\triangle ABC$ be a triangle with sides measuring $1/2$, 1 and $\sqrt{3}/2$. Find the radius of the circle inscribed in the triangle, i.e., of the circle inside the triangle and tangent to its three sides.

(a) $\frac{\sqrt{3} + 1}{2}$  (b) $\frac{\sqrt{3} + 1}{3}$  (c) $1/4$  (d) $\frac{\sqrt{3}}{4}$  (e) none of these
17. Ten couples must sit around a round table in such a way that every couple sits together. How many possible seating arrangements are there if we only regard who sits next to whom? (In other words, rotating everyone a fixed number of seats does not yield a different configuration.)

(a) \( \binom{20}{10} \)
(b) \( 9! \cdot 2^{10} \)
(c) \( \binom{20}{2}^{10} \)
(d) \( 10! \cdot 2^{19} \)
(e) none of these

18. The rhombus \( ABCD \) shown below has sides measuring 1, and \( M \) and \( N \) are the midpoints of the sides \( AB \) and \( BC \) respectively.

If the length of the segment \( MN \) is \( \sqrt{14}/4 \), then the length of the segment \( DM \) is:

(a) \( \frac{\sqrt{2}}{2} \)  
(b) \( \frac{3\sqrt{2}}{4} \)  
(c) \( \frac{\sqrt{2}}{4} \)  
(d) \( \sqrt{2} \)  
(e) none of these

19. The sum of the first one hundred odd positive integers which are not divisible by 3 is:

(a) 15000  
(b) 7400  
(c) 7600  
(d) 1633  
(e) none of these
20. A circular garden of diameter 12 feet is cut by a path 3 feet wide, one side of which goes through the center of the garden.

The garden area which is not part of the path (in square feet) is:

(a) $20\pi - 5\sqrt{3}$

(b) $30\pi - 9\sqrt{3}$

(c) $30\pi - 6\sqrt{3}$

(d) $20\pi$

(e) none of these

21. We have two urns, urn 1 and urn 2. Urn 1 contains two black balls and one white ball. Urn 2 contains two black balls and two white balls. An urn is chosen at random and one ball is removed (and not replaced). Then, another ball is removed following the same process. What is the probability that both balls removed are black?

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) $\frac{9}{24}$

(d) $\frac{7}{24}$

(e) none of these
22. Suppose that the equation \( x^3 - px^2 = q^m \), where \( p \) and \( q \) are positive real numbers with \( q \neq 1 \) and \( m \) is a positive integer, has three positive real solutions \( a, b \) and \( c \). Then,

\[
\log_q (abc(a^2 + b^2 + c^2)^{a+b+c})
\]

is equal to:

(a) \( 2m - p \log_q(p) \)
(b) \( m + 2p \log_q(p) \)
(c) \( 1 \)
(d) \( m + p \log_q(p) \)
(e) none of these

23. The last digit (from left to right) of \( 2012^{2012} \) is:

(a) 8  (b) 4  (c) 2  (d) 6  (e) none of these

24. Three integers from the set \{1, 2, \ldots, 9, 10\} are randomly chosen (with repetitions allowed). What is the probability that the sum of the three is less than or equal to 9?

(a) 12.4%  (b) 8.4%  (c) 29.5%  (d) 16.2%  (e) none of these

25. The number of consecutive zeros at the end (from left to right) of the decimal representation of \( 2012! \) is:

(a) 100  (b) 301  (c) 501  (d) 2012  (e) none of these