1) Ten percent of the students taking a math exam fail the exam. Forty percent of the students taking the exam are boys and fifteen percent of these boys fail the exam. Twenty girls fail the exam. How many total students took the exam?

2) In how many integers \( n \) with \( 1,000 \leq n \leq 9,999 \) does the digit 3 occur?

3) A rectangular piece of paper with an area of 400 cm\(^2\) is shaped so that when it is folded in half, each half is a rectangle similar to the original sheet. Find exact expressions for the length and width of the original sheet of paper.

4) Let \( m \) be a positive integer and \( P(X) = X^3 + mX + 1 \). Show that not all of the roots of \( P(X) \) are real numbers.

5) Let \( x \) and \( y \) be positive integers such that \( y \) is obtained from \( x \) by permuting digits (for example: \( x = 7315 \) and \( y = 1357 \), or \( x = 024 \) and \( y = 204 \)). Show that \( x - y \) is divisible by 9.

6) A disc of diameter 1 is inside a square with base of length \( a > 1 \). A dot is affixed to the circumference of the disc and another dot to the base of the square as in the picture below so that the dots line up initially. The disc rolls around the inside edge of the square in a counterclockwise direction in such a way that there is no slippage. Determine conditions on the length of the base \( a \) so that the dots line up after a specified number of circuits \( N \) around the inside of the square.
7) Two factories, A and B, are located on opposite sides of a straight river of constant width as in the picture below. Where should we build a bridge, perpendicular to the river banks, so that the highway connecting the bridge and the two factories is as short as possible?

8) Let $a_1, a_2, a_3,$ and $a_4$ be integers. Show that the product:

$$(a_1 - a_2)(a_1 - a_3)(a_1 - a_4)(a_2 - a_3)(a_2 - a_4)(a_3 - a_4)$$

is divisible by 12.