1. A 10 inches by 10 inches square is laid down on the top of another square of the same size and a vertex of the square on the top is at center of the square at bottom (see Figure 1). Suppose the top square is movable, as long as its vertex remains in the center of the other square. What is the largest possible area where two squares overlap?

(a) 20 square inches  
(b) 25 square inches  
(c) 30 square inches  
(d) 50 square inches  
(e) None of the above

2. Let ABCD be a square located on a Cartesian coordinate system with A at the origin, B on the y axis, and D on the x axis. P is a point in the interior of the square that is 3 units from A, 5 units from D, and 7 units from B (see Figure 2). What is the length of a side of the square (accurate to the second decimal place) ?
(a) 7.2 units  
(b) 7.4 units  
(c) 7.6 units  
(d) 7.8 units  
(e) None of the above

3. Let \( \{a_1, a_2, \ldots, a_{99}\} \) and \( \{b_1, b_2, \ldots, b_{99}\} \) be nonzero real numbers. How many multiplications one need to perform in order to evaluate \( \sum_{i=1}^{99} (a_ib_1 + a_ib_2 + \cdots + a_ib_i) \)?

(a) 99  
(b) 3520  
(c) 4950  
(d) 9801  
(e) None of the above

4. How many different right triangles with integer side lengths exist with a leg equal to 15?

(a) none  
(b) 1  
(c) 2  
(d) 3  
(e) 4

5. Find the value of \( n \) if \( \log_2(\log_3(\log_9 3^n)) = 2 \).

(a) 64  
(b) 81  
(c) 128  
(d) 162  
(e) none of the above

6. Find all \( x \) such that \( (\log_x 2)(\log_x 8) + 6 = 3 \log_x 8 \).

(a) \( x = 1, 2 \)  
(b) \( x = 2, \sqrt{2} \)  
(c) \( x = 2, -\sqrt{2}, \sqrt{2} \)  
(d) \( x = -2, 2, -\sqrt{2}, \sqrt{2} \)  
(e) none of the above

7. Let the function \( f(x) \) be defined recursively by

\[
f(x + 1) = \begin{cases} 
    f(x) + 3 & \text{if } f(x) \leq 100, \\
    f(x) - 2 & \text{if } f(x) > 100.
\end{cases}
\]

If \( f(1) = 50 \), what is the function value at \( x = 205 \)?

(a) 80  
(b) 102  
(c) 105  
(d) 126  
(e) none of the above
8. The area of the region bounded by the graph of the equation $|x + y| + |x - y| = 4$ is
   (a) 16 square units    (d) 4 square units
   (b) 12 square units    (e) none of the above
   (c) 8 square units

9. A point is chosen at random from within a circular board of radius 5 inches. What is the probability that the point will be closer to the center of the board than to the boundary of the region?
   (a) $\frac{1}{2}$    (c) $\frac{1}{8}$
   (b) $\frac{1}{4}$    (d) $\frac{1}{25}$
   (e) none of the above

10. In a sequence of numbers, every number except the first two is the sum of the two numbers before it. If the first number is 1, and the tenth number is 201, what is the second number?
    (a) $\frac{5}{17} \frac{2}{17}$    (c) $\frac{5}{17} \frac{4}{17}$
    (b) $\frac{5}{17} \frac{3}{17}$    (d) $\frac{5}{17} \frac{5}{17}$
    (e) none of the above

11. The shortest distance from the point $(1, 3)$ to all points on the straight line $2x + y = 1$ is
    (a) $\frac{4\sqrt{5}}{5}$ units    (c) $\frac{5}{4}$ units
    (b) $\frac{4\sqrt{3}}{5}$ units    (d) $\frac{6}{5}$ units
    (e) none of the above

12. Suppose $y > 0$, $x > y$ and $z \neq 0$, the inequality which is always correct is
    (a) $x + z > y - z$    (d) $xz^2 > yz^2$
    (b) $xy > yz$
    (c) $\frac{x}{z} > \frac{y}{z}$    (e) none of the above
13. The number of solutions to the linear system is

\[
\begin{align*}
    x_1 - 2x_2 + 5x_3 &= 2 \\
    4x_1 - 5x_2 + 8x_3 &= 1 \\
    -3x_1 + 3x_2 - 3x_3 &= 1
\end{align*}
\]

(a) none  
(b) 1  
(c) 2  
(d) more than 2, but finitely many  
(e) infinitely many

14. The largest integer that is less than \( \sqrt{2^{100} + 10^{10}} \) is

(a) \( 2^{50} \)  
(b) \( 2^{50} + 1 \)  
(c) \( 2^{50} + 10 \)  
(d) \( 2^{50} + 100 \)  
(e) \( 2^{50} + 10^5 \)

15. Use absolute value notation to describe: 6 is at most 3 units from \( x \).

(a) \( |x - 3| < 6 \)  
(b) \( |x - 3| \leq 3 \)  
(c) \( |x - 6| \geq 3 \)  
(d) \( |x - 3| \geq 6 \)  
(e) none of the above

16. The coefficient of the term containing \( x^6y^7 \) in \( (x + 2y)^{12} \) is

(a) 0  
(b) 64  
(c) 128  
(d) \( 2^{12} \)  
(e) none of the above

17. Find the number of units that produce a maximum revenue, \( R = 95x - 0.1x^2 \), where \( R \) is the total revenue in dollars and \( x \) is the number of units sold.

(a) 716 units  
(b) 642 units  
(c) 550 units  
(d) 475 units  
(e) none of the above

18. The possible rational zeros of the polynomial \( p(x) = 3x^5 + 7x^3 - 3x^2 + 2 \) are

(a) \( \pm \frac{2}{3}, \pm \frac{3}{2}, \pm 2, \pm 3 \)  
(b) \( \pm \frac{1}{3}, \pm \frac{3}{2}, \pm 1, \pm 2 \)  
(c) \( \pm \frac{3}{2}, \pm \frac{1}{2}, \pm 3, \pm 1 \)  
(d) \( \pm \frac{3}{2}, \pm \frac{2}{3}, \pm \frac{1}{2}, \pm \frac{1}{3} \)  
(e) none of the above
19. Two squares are inscribed in a semicircle as shown in Figure 3. If the area of the smaller square is 15 square units, what is the area of the larger square?

![Figure 3:](image)

(a) 30 square units  
(b) 40 square units  
(c) 55 square units  
(d) 60 square units  
(e) none of the above

20. It is known that \( \log_{10} 5 = 0.69897 \), correct to five digits. How many digits are there in the decimal representation of \( 5^{100} \)?

(a) 57  
(b) 70  
(c) 83  
(d) 100  
(e) none of the above

21. Suppose \( \frac{5}{4} < x < \frac{11}{4} \), find the value of \( \sqrt{x^2 - 2x + 1} + \sqrt{x^2 - 6x + 9} \)

(a) 2  
(b) 3  
(c) \( 2x - 4 \)  
(d) \( 4x - 2 \)  
(e) none of the above

22. Mary and Lucy are among 6 girls who are seated at random in a row, the probability that exactly 2 girls are seated between them is

(a) \( \frac{1}{8} \)  
(b) \( \frac{1}{6} \)  
(c) \( \frac{1}{5} \)  
(d) \( \frac{1}{4} \)  
(e) none of the above
23. What is area of the region common to two unit circles whose centers are \( \sqrt{2} \) apart?

(a) \( \frac{1}{2} \left( 1 - \frac{\pi}{4} \right) \)  
(b) \( \frac{\pi}{2} \)  
(c) \( 1 - \frac{\pi}{4} \)

(d) \( \frac{\pi}{2} - 1 \)

24. The number \( x, y \) and \( z \) satisfy \(|x + 3| + |y + 4| + |z - 7| = 1\). Which of the following could be \(|x + y + z|\)?

(a) 0  
(b) 3  
(c) 7

(d) 10  
(e) none of the above

25. How many solutions do the trigonometric equation \( \frac{\sin x}{1 + \cos x} = 1 \) have in the interval \([0, 2\pi]\)?

(a) 0  
(b) 2  
(c) 4

(d) infinitely many  
(e) none of the above