Problem Solving

One goal of this course is to for you to be able to solve problems of a type that you have never seen before. You should learn how to start thinking about an unfamiliar problem, how to use several different strategies to reach an answer, and how to check your answer to see if it is reasonable.

George Polya suggests a four-step approach to solving problems:

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Look back and check your answer.

Some strategies that you might use when devising or carrying out the plan:

- Draw a picture.
- Build a model.
- Guess and check.
- Solve a similar but simpler problem.
- Make a table or chart.
- Find a pattern.
- Work backwards.
- Use technology.
- Break the problem up into smaller problems.
- Rephrase in terms of a problem you’ve already solved.
- Combine strategies.

Homework Problems

1. If you draw 15 dots around a circle, how many lines can you draw connecting pairs of dots?

2. You are selling lamps at a craft fair. You are required to collect 9.25% sales tax on all of your sales. If you want a lamp to cost $20 including the tax, what pre-tax price should you charge? Round to the nearest cent.
3. Three clubs were meeting on the same day at the same time: cheerleaders, pep club, and student council. At each meeting not all club members were present because some students belong to more than one club. Half of the student council members and all the cheerleaders belong to the pep club. Two students are members of all three clubs. If there are 24 student council members, 6 cheerleaders, and 40 pep club members, how many students belong only to the pep club?

4. You drive to Asheville at an average speed of 60 miles per hour. There is construction on the interstate and an accident slowing up traffic on your way back, and you average a speed of 30 miles an hour on your trip back to Knoxville. What was your average speed for the round trip? (Hint: it doesn’t matter how far it is to Asheville.)

5. You are working at an inn about an hour south of here. During the busy season, your inn is booked up with tourists, and your boss has decided to run a shuttle to the airport. Your boss wants a shuttle to leave the inn every 10 minutes. You have been put in charge of arranging this with the van company. The van company tells you that it will take 52.5 minutes to drive from the inn to the airport and that you should allow 7.5 minutes for loading and unloading at each stop. How many vans will you need?

6. The zoo is ordering birdseed. The keeper of the bird cages knows that two cockatoos will eat 2 pounds of seed every 2 weeks, three parrots will eat 3 pounds of seed every 3 weeks, and 4 macaws will eat 4 pounds of seed every 4 weeks. How much birdseed should the zoo order for 12 cockatoos, 12 parrots, and 12 macaws for 12 weeks?

7. If you have a 5-quart pail and a 9-quart pail (and no other containers), how can you go to a water source and bring back exactly 7 quarts of water?

8. A restaurant offers 15 different toppings for its pizzas. How many ways can you select two toppings?

9. If a hundred chickens eat a hundred bushels of grain in a hundred days, how many bushels will ten chickens eat in ten days? And if, on the average, one and a half of these chickens lay an egg and a half in a day and a half, how many days will it take a chicken to lay one and a half dozen eggs?

10. In a cross-country run, Sven placed exactly in the middle among all participants. Dan placed lower (i.e. did worse than Sven), in tenth place, and Lars placed sixteenth. How many runners took part in the race?

11. You have a strip of paper that is two-thirds of a meter long. However, you need a strip exactly half a meter long. Must you have to ruler to cut off such a length?
In Class Problems

1. You are organizing a tournament involving 15 teams. How many games will have to be played so that every team has played each other team once?

2. (Problem 4 from page 29) Fifty-six biscuits are to be fed to 10 pets; each pet is either a cat or a dog. Each dog is to get six biscuits, and each cat is to get five. How many dogs are there?

3. On the final day of his close-out sale, a merchant hastily disposed of two lamps at the bargain price of twelve dollars apiece. He estimated that he must have made some profit on the combined transactions since he made a 25 percent profit on one and only took a 20 percent loss on the other. How much of a profit (or loss) did he make?

4. A farmer looks out into the barnyard and sees the pigs and chickens. He says to his daughter, “I count 169 heads and 398 feet.” How many pigs and how many chickens are there?

5. A restaurant offers five different toppings for its pizzas. How many ways can you select two toppings? three toppings? What if there were seven different toppings?

6. It takes a blacksmith five minutes to put on a horseshoe. How long would it take for eight blacksmiths to shoe ten horses? A horse can not stand on two legs — it can only have one shoe put on at a time.

7. (Condensed version of problem 14 on page 32) Five couples were present at a party (10 people total, including the host and hostess). Some people shook hands with other guests. No one shook hands with themselves and no one shook hands with his or her own spouse. A guest might not shake anyone’s hand or might shake as many as eight other people’s hands. At the end of the party the host asked the other nine people how many hands they shook. One person didn’t shake any hands, someone else shook one hand, someone else shook two hands, someone else shook three hands, and so forth to the last person who shook eight hands. Determine the number of hands that the hostess shook.