1. Describe in words the region in $\mathbb{R}^3$ represented by the following inequalities. Give enough information to completely describe the region.

(a) $z > -1$
(b) $x^2 + y^2 + (z - 2)^2 \leq 4$

10 pts (a) Set of points strictly above the plane parallel to the $xy$-plane and hitting the $z$-axis at $z = -1$.

10 pts (b) The points on and inside of the sphere of radius 2 centered at $(0, 0, 2)$. 

2. Consider the vectors \( \mathbf{u} = (1, 2, 0) \) and \( \mathbf{v} = (0, 1, 1) \).

(a) Calculate
i. \( 5\mathbf{u} - \mathbf{v} \)
ii. \( \mathbf{u} \cdot \mathbf{v} \)
iii. \( \mathbf{u} \times \mathbf{v} \)

(b) What is the angle between \( \mathbf{u} \) and \( \mathbf{v} \)?

\[
\begin{align*}
(2) & \quad 5\langle 1, 2, 0 \rangle - \langle 0, 1, 1 \rangle \\
& = \langle 5, 10, 0 \rangle - \langle 0, 1, 1 \rangle \\
& = \langle 5, 9, -1 \rangle
\end{align*}
\]

\[
5\text{pts (i)} \quad \langle 1, 2, 0 \rangle \cdot \langle 0, 1, 1 \rangle \\
= 0 \cdot 1 + 2 \cdot 1 + 0 \cdot 1 = 2
\]

\[
5\text{pts (ii)} \quad \begin{vmatrix} i & j & k \\ 1 & 2 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \langle 2, 1 - 0, -(1 \cdot 1 - 0 \cdot 0) \rangle, 1 \cdot 1 - 0 \cdot 2 \\
= \langle 2, -1, 1 \rangle
\]

\[
5\text{pts (iii)} \quad |\mathbf{u}| = \sqrt{1^2 + 2^2} = \sqrt{5} \\
|\mathbf{v}| = \sqrt{1^2 + 1^2} = \sqrt{2}
\]

\[
\cos \Theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}| |\mathbf{v}|} = \frac{2}{\sqrt{5} \sqrt{2}} = \frac{\sqrt{10}}{10} = \frac{\sqrt{10}}{5}
\]

\[\Theta = \cos^{-1} \left( \frac{\sqrt{10}}{5} \right)\]
3. Find the vector equation of a line through the points \((-2, 5, 0)\) and \((4, 11, -1)\).

\[
P = (-2, 5, 0) \quad Q = (4, 11, -1)
\]

\[
\mathbf{V} = \overrightarrow{PQ} = \langle 4 - (-2), 11 - 5, -1 - 0 \rangle
\]

\[
= \langle 6, 6, -1 \rangle = \text{VECTOR PARALLEL TO LINE}
\]

\[
\mathbf{r}_0 = \langle -2, 5, 0 \rangle = \text{POSITION VECTOR OF A POINT ON THE LINE}
\]

\[
\left( \mathbf{r}_0 = \langle 4, 11, -1 \rangle \text{ ALSO WORKS.} \right)
\]

\[
\mathbf{r} = \mathbf{r}_0 + t \mathbf{V}
\]

\[
= \langle -2, 5, 0 \rangle + t \langle 6, 6, -1 \rangle
\]

\[
= \langle -2 + 6t, 5 + 6t, -t \rangle.
\]

\[
20p+5
\]
4. Find the equation of a plane through the point \( (1, -1, 1) \) and which contains the line with vector equation \( \mathbf{r} = (t, \frac{1}{2}t, \frac{1}{3}t) \).

\[
\mathbf{r} = t\begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} t \\ \frac{1}{2}t \\ \frac{1}{3}t \end{pmatrix}.
\]

We must find the normal vector:

\[
\mathbf{n} = t\begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
\]

\( \mathbf{n} \) is parallel to the line, so \( \mathbf{n} \) is a vector in the plane.

The tip of \( \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \), \( Q = (0, 0, 0) \), is a point in the plane.

So \( \mathbf{PQ} = \begin{pmatrix} 0-1 \\ 0-(-1) \\ 0-1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \) is another vector in the plane.

\( \mathbf{n} \times \mathbf{PQ} \) is a normal vector:

\[
\begin{vmatrix}
2 & \frac{1}{2} & \frac{1}{3} \\
1 & \frac{1}{2} & \frac{1}{3} \\
-1 & 1 & 1
\end{vmatrix} = \begin{pmatrix} -\frac{1}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \end{pmatrix} = \begin{pmatrix} -1/3 \\ -1/3 \\ -1/3 \end{pmatrix}.
\]
So the equation for the plane is

\[ a(x-x_0) + b(y-y_0) + c(z-z_0) = 0 \]

where \( <a, b, c> \) is the normal vector
\( (x_0, y_0, z_0) \) is a point on the plane

\[-\frac{5}{6}(x-0) + \frac{2}{3}(y-0) + \frac{3}{2}(z-0) = 0\]

\[-\frac{5}{6}x + \frac{2}{3}y + \frac{3}{2}z = 0.\]

(One can multiply by 6 to clear the denominators:

\[-5x + 4y + 9z = 0.\)
5. Match the functions with their graph.

30. (a) $z = 3x^2 + y$  
(b) $z = y^2 - 3x^2$  
(c) $z = \sin \sqrt{x^2 + y^2}$  
(d) $z = \sin x \sin y$  
(e) $z = \frac{3}{2}y^2 + x^2$  
(f) $z = y(y^2 - 3x^2)$

The $y=0$ trace distinguishes most of these. See next page!
\[ z = \text{C} + \text{d} \]
\[ y = 0 \] traces

(a) \( z = 3x^2 \) \( \text{CORF (PARABOLA)} \)
(b) \( z = -3x^2 \) \( \text{D (UPSIDE DOWN PARABOLA)} \)
(c) \( z = \sin|x| \) \( \text{A (OSCILLATING FUNCTION)} \)
(d) \( z = 0 \) \( \text{EITHER B OR E} \)
(e) \( z = x^2 \) \( \text{CORF (PARABOLA)} \)
(f) \( z = 0 \) \( \text{EITHER B OR E} \)

You can determine these by seeing how the \( xz \)-plane intersects the graph.

Narrow it further using \( x = 0 \),

(a) \( z = y \) \( \text{LINE AT 45° ANGLE MUST BE F} \)
(b) \( z = \sin y \) \( \text{MUST BE B} \).
(c) \( z = \frac{3}{2}y^2 \) \( \text{MUST BE C} \)
(f) \( z = y^3 \) \( \text{MUST BE E} \).
6. **Bonus Question:** The line \( L \), with vector equation, \((x, y, z) = (-t, t, 4 - t)\) lies in the plane \( x + 2y + z = 4 \). The point \( P(1, 1, 1) \) also lies in this plane. Your task is to find the vector equation of a line which passes through \( P \), is orthogonal to \( L \), and lies in the given plane.

Find the equation of the dotted line!

\[ L \text{ has equation } \vec{r} = t\langle -1, 1, -1 \rangle + \langle 0, 0, 4 \rangle \]

\[ \vec{n} = \langle 1, 2, 1 \rangle \]

**Observation** \( \vec{n} \times \vec{v} \) is parallel to dotted line!

\[ \vec{n} \times \vec{v} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ -1 & 1 & -1 \end{vmatrix} = \langle 2(-1) - 1 \cdot 1, -(1 \cdot -1 - (-1) \cdot 1), 1 \cdot 1 - (-1) \cdot 2 \rangle = \langle -3, 0, 3 \rangle \]

Equation of the line \( \vec{p} \)

Dotted

\[ \vec{p} = t\langle -3, 0, 3 \rangle + \langle 1, 1, 1 \rangle \]

Position

\[ \vec{p} = \langle -3t + 1, 1, 3t + 1 \rangle \]