1. For the closed curve in the (x,y) plane with parametric equations:

\[ x(t) = R \cos^3 t, \quad y(t) = R \sin^3 t, \quad t \in [0, 2\pi] \]

(i) Find the total length of the curve (given: \( \int_{\pi/2}^{0} \cos t \sin t \, dt = 1/2 \))

(ii) Find the area of the region enclosed by the curve (given: \( \int_{0}^{2\pi} \cos^2 t \sin^2 t \, dt = \pi/4. \))

2. Consider the region \( D \) bounded by the x-axis and the graph of the function:

\[ y = f(x) = H(1 - x^4), \quad x \in [-1, 1]. \]

(a) Find the area of \( D \);

(b) Find the y-coordinate of the center of mass of \( D \);

(c) Find the volume of the solid obtained by rotating \( D \) about the y-axis.

3. Find the center of mass of the region in the first quadrant sketched below and the volume of the region obtained by rotating it about the y-axis (not necessarily in this order.) You may use Pappus’ theorem.

4. The time spent waiting in line at a certain bank is modelled by an exponential probability density function with mean 8 minutes.

(a) What is the probability that the customer is served in the first three minutes?

(b) What is the median waiting time?