Math231 Quiz #10 Fall 2023 Name:

1. Find the inverse Laplace transform

$$\mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}e^{-3s}\Big)$$

Solution.

$$\mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}e^{-3s}\Big) = u(t-3)\mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}\Big)(t-3)$$

On the other hand,

$$\mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}\Big) = \mathcal{L}^{-1}\Big(\frac{1}{s} - \frac{s}{s^2+1}\Big) = \mathcal{L}^{-1}\Big(\frac{1}{s}\Big) - \mathcal{L}^{-1}\Big(\frac{s}{s^2+1}\Big) = 1 - \cos t$$

Hence,

$$\mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}e^{-3s}\Big) = u(t-3)\big(1-\cos(t-3)\big)$$

2. Solve the initial value problem $y'' + y = u(t-3)$, $y(0) = y'(0) = 0$ by the method of Laplace transform.

Solution. Taking Laplace transform

$$s^2\mathcal{L}(y) + \mathcal{L}(y) = \mathcal{L}\big(u(t-3)\big) = \frac{1}{s}e^{-3s}$$

Thus,

$$\mathcal{L}(y) = \frac{1}{s(s^2+1)}e^{-3s}$$

Hence

$$y = \mathcal{L}^{-1}\Big(\frac{1}{s(s^2+1)}e^{-3s}\Big) = u(t-3)\big(1-\cos(t-3)\big)$$