Math 231 **Quiz #9** **Fall 2023** **Name:**

1. Find the inverse Laplace transform

$$\mathcal{L}^{-1}\Big(\frac{-s^2+2s+11}{(s-1)(s+5)(s+1)}\Big)$$

Solution. We have the decomposition

$$\frac{-s^2+2s+11}{(s+1)(s+5)(s+1)} = \frac{1}{s-1} - \frac{1}{s+5} - \frac{1}{s+1}$$

Hence,

$$\mathcal{L}^{-1}\Big(\frac{-s^2+2s+11}{(s+1)(s+5)(s+1)}\Big) = \mathcal{L}^{-1}\Big(\frac{1}{s-1}\Big) - \mathcal{L}^{-1}\Big(\frac{1}{s+5}\Big) - \mathcal{L}^{-1}\Big(\frac{1}{s+1}\Big) = e^t - e^{-5t} - e^{-t}$$

2. Solve the initial value problem $y''+6y'+5y = 12t^t$, $y(0) = -1$, $y'(0) = 7$ using the method of Laplace transform.

Solution. Take Laplace transform

$$\mathcal{L}(y'') + 6\mathcal{L}(y') + 5\mathcal{L}(y) = 12\mathcal{L}(e^t) = \frac{12}{s-1}$$

$$(s^2\mathcal{L}(y) + s - 7) + 6(s\mathcal{L}(y) + 1) + 5\mathcal{L}(y) = \frac{12}{s-1}$$

Solving for $\mathcal{L}(y)$:

$$\mathcal{L}(y) = \frac{1}{s^2+6s+5}\Big(1 - s + \frac{12}{s-1}\Big) = \frac{-s^2+2s+11}{(s+1)(s+5)(s+1)}$$

Hence,

$$y = \mathcal{L}^{-1}\Big(\frac{-s^2+2s+11}{(s+1)(s+5)(s+1)}\Big) = e^t - e^{-5t} - e^{-t}$$