

1. Find the Laplace transform $\mathcal{L}(te^t \sin 2t)$

Solution.

$$\mathcal{L}(e^t \sin 2t) = \mathcal{L}(\sin 2t)(s - 1) = \frac{2}{(s - 1)^2 + 4}$$

Hence

$$\mathcal{L}(te^t \sin 2t) = -\frac{d}{ds} \mathcal{L}(e^t \sin 2t) = -\frac{d}{ds} \frac{2}{(s - 1)^2 + 4} = \frac{4(s - 1)}{((s - 1)^2 + 4)^2}$$

2. Find the Laplace transform $\mathcal{L}(\sin^2 t)$

Solution. $\sin^2 t = \frac{1 - \cos 2t}{2}$

$$\mathcal{L}(\sin^2 t) = \frac{1}{2} \mathcal{L}(1) - \frac{1}{2} \mathcal{L}(\cos 2t) = \frac{1}{2s} - \frac{1}{2} \frac{s}{s^2 + 4} \quad (s \geq 0)$$