Math 231 **Quiz #7** **Fall 2023** **Name:**

A 3-kg mass is attached to a spring with stiffness $k = 48N/m$. The mass is displaced 1/2 m to the left of the equilibrium point and given a velocity of $2m/sec$ to the right. The damping force is negligible.

1. Neglecting external force, find the equation of the motion along with amplitude A and the initial angle φ.

Solution. Let $y(t)$ be the displacement of the mass at the time t. We have

$$3y" + 48y = 0, \quad y(0) + -\frac{1}{2} \text{ and } y'(0) = 2$$

Soving $3r^2 + 48 = 0$ we have $r \pm 4i$. Therefore $y = A\sin(4t + \varphi)$ and therefore $y' = 4A\cos(4t + \varphi)$. Let $t = 0$: $A\sin\varphi = -\frac{1}{2}$ and $4A\cos\varphi = 2$. Divide the first equation by the second equation: $\tan\varphi = -1$. So $\varphi = -\pi/4$. From the second equation, $A\cos\varphi = \frac{1}{2}$. Hence

$$A^2 = A^2\cos^2\varphi + A^2\sin^2\varphi = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ Therefore } A = \frac{1}{\sqrt{2}}$$

Finally, $y = \frac{1}{\sqrt{2}}\sin(4t - \frac{\pi}{4})$

2. With the external force $F(t) = \cos\omega t$, find the external frequency ω that leads to resonance and set-up a special solution y_p to the equation of the motion in this case **without identifying the constants**.

Solution. The resonance frequency $\omega = 4$.

$$y_p = t(C_1\sin 4t + C_2\cos 4t)$$