**Math 231** **Quiz #6** **Fall 2023** **Name:**

1. Find the general solution for $t^2y'' + 4ty' + 2y = 0$.

**Solution.** Consider $r^2 + (4-1)r + 2 = 0$ or $r^2 + 3r + 2 = 0$. We have that $r_1 = -1$ and $r_2 = -2$. The general solution is: $y = C_1t^{-1} + C_2t^{-2}$.

2. Find the general solution for $t^2y'' + 4ty' + 2y = t^{-1}$.

**Solution.** We first find a special solution $y_p$. By variation of parameter we set $y_p = v_1(t)t^{-1} + v_2(t)t^{-2}$. So we have

$$\begin{cases} v_1'(t)t^{-1} + v_2'(t)t^{-2} = 0 \\ -v_1'(t)t^{-2} - 2v_2'(t)t^{-3} = \frac{t^{-1}}{t^2} = t^{-3} \end{cases} \quad \text{or} \quad \begin{cases} v_1'(t)t^{-1} + v_2'(t)t^{-2} = 0 \\ -v_1'(t)t^{-1} - 2v_2'(t)t^{-2} = t^{-2} \end{cases}$$

Adding two equations together, $-v_2'(t)t^{-2} = 1$ or $v_2'(t) = -1$. Bring it to the first equation, $v_1'(t) = t^{-1}$. Hence, $v_1(t) = \ln t$ and $v_2(t) = -t$. Thus, $y_p = t^{-1}\ln t - t^{-1}$. Finally, the general solution is

$$y = C_1t^{-1} + C_2t^{-2} + t^{-1}\ln t - t^{-1}$$