

1. Find the general solution to $y'' + 4y = 0$.

Solution. Solve $r^2 + 4 = 0$ we have $r_1 = 2i$ and $r_2 = -2i$. The general solution is

$$y = C_1 \cos 2t + C_2 \sin 2t$$

2. Find the general solution to $y'' + 4y = 16 \sin 2t$.

Solution. Set up special solution

$$y_p = t(A \sin 2t + B \cos 2t)$$

$$y'_p = (A \sin 2t + B \cos 2t) + t(2A \cos 2t - 2B \sin 2t)$$

$$\begin{aligned} y''_p &= (2A \cos 2t - 2B \sin 2t) + (2A \cos 2t - 2B \sin 2t) + t(-4A \sin 2t - 4B \cos 2t) \\ &= (4A \cos 2t - 4B \sin 2t) - 4t(A \sin 2t + B \cos 2t) \end{aligned}$$

$$y_p'' + 4y_p = 4A \cos 2t - 4B \sin 2t$$

Thus, $4A = 0$ and $-4B = 16$. So $A = 0$ and $B = -4$. Therefore, $y_p = -4t \cos 2t$. The general solution is

$$y = C_1 \cos 2t + C_2 \sin 2t - 4t \cos 2t$$