

A red wine initially at $10^\circ C$ warms up to $15^\circ C$ in a room of temperature $23^\circ C$ in 10min. How warm will the wine be if left for 20 min?

Solution. Let $T = T(t)$ be the wine temperature at the time t

$$\begin{cases} \frac{dT}{dt} = K(23 - T) \\ T(0) = 10 \quad \text{and} \quad T(10) = 15 \end{cases}$$

By variable separation

$$\frac{dT}{23 - T} = K dt, \quad 23 - T = C e^{-Kt}, \quad T = 23 - C e^{-Kt}$$

Let $t = 0$, $C = 13$. So $T = 23 - 13e^{-Kt}$

Let $t = 10$: $K = \frac{1}{10} \ln \frac{13}{8}$. Hence,

$$T = 23 - 13 \exp \left\{ -\frac{t}{10} \ln \frac{13}{8} \right\}$$

Finally,

$$T(20) = 23 - 13 \exp \left\{ -\frac{20}{10} \ln \frac{13}{8} \right\} = 23 - 13 \times \left(\frac{8}{13} \right)^2 \approx 18^\circ C$$