

1. Solve the initial problem $\frac{dy}{dx} = x^2(1+y)$, $y(0) = 3$.

Solution.

$$\frac{dy}{1+y} = x^2 dx, \quad \ln|1+y| = \frac{1}{3}x^3 + \ln C, \quad |1+y| = C \exp\left\{\frac{1}{3}x^3\right\}, \quad y = C \exp\left\{\frac{1}{3}x^3\right\} - 1$$

Let $x = 0$: $3 = C - 1$. $C = 4$.

$$y = 4 \exp\left\{\frac{1}{3}x^3\right\} - 1$$

2. Obtain the general solution for $\frac{dy}{dx} - \frac{y}{x} = xe^x$

Solution.

$$y = \exp\left\{\int \frac{1}{x} dx\right\} \left[\int \exp\left\{-\int \frac{1}{x} dx\right\} x e^x dx + C \right] = x \left[\int \frac{1}{x} \cdot x e^x dx + C \right] = x(e^x + C)$$