Math 231 **Quiz #2** **Fall 2023** **Name:**

1. Solve $(\cos x \cos y + 2x)dx - (\sin x \sin y + 2y)dy = 0$.

Solution. Let $M(x, y) = \cos x \cos y + 2x$ and $N(x, y) = -(\sin x \sin y + 2y)$.

$$\frac{\partial M}{\partial y} = -\cos x \sin y \quad \textit{and} \quad \frac{\partial N}{\partial x} = -\cos x \sin y$$

They are equal. So the equation is exact. We now solve

$$\frac{\partial F}{\partial x} = \cos x \cos y + 2x \quad \textit{and} \quad \frac{\partial F}{\partial y} = -(\sin x \sin y + 2y)$$

Take anti-derivative in the first equation: $F = \sin x \cos y + x^2 + C(y)$. Take the partial derivative:

$$\frac{\partial F}{\partial y} = -\sin x \sin y + C'(y)$$

So we have $C'(y) = -2y$ or $C(y) = -y^2$. Hence, $F = \sin x \cos y + x^2 - y^2$. The general solution is:

$$\sin x \cos y + x^2 - y^2 = C$$

2. Solve $\frac{dy}{dx} = (x + y + 2)^2$.

Solution. Let $v = x + y$.

$$\frac{dv}{dx} = 1 + \frac{dy}{dx} = 1 + (x + y + 2)^2 = 1 + (v + 2)^2$$

$$\frac{dv}{1 + (v + 2)^2} = dx$$

$$\tan^{-1}(v + 2) = x + C \quad \text{or} \quad v = \tan(x + C) - 2$$

The solution is

$$x + y = \tan(x + C) - 2 \quad \text{or} \quad y = \tan(x + C) - x - 2$$