Finally we note that a group can satisfy both chain conditions on normal subgroups and have infinitely many normal subgroups.

THEOREM 4. *A group G satisfying both chain conditions on normal subgroups has infinitely many normal subgroups if and only if there is a normal subgroup N of G, a group H and a simple H-module A with* $\mathrm{Aut}_H(A)$ *infinite such that G/N is an extension of $A \times A$ by H inducing the given operation of H on A.*

An example of such a group is provided by the semidirect product of $R^3 \oplus R^3$ by SO_3 where R^3 is Euclidean 3-space and SO_3 is its group of rotations.

A. W. HALES: DEPARTMENT OF MATHEMATICS, UNIVERSITY OF CALIFORNIA, LOS ANGELES, CALIFORNIA 90024.

R. J. NUNKE: DEPARTMENT OF MATHEMATICS, UNIVERSITY OF WASHINGTON, SEATTLE, WASHINGTON 98109.

AUTOMORPHISMS OF p-ADIC NUMBER FIELDS

C. G. WAGNER

In 1933 F. K. Schmidt [**4**, p. 3] proved a theorem which characterizes those fields which are complete with respect to at least two non-trivial inequivalent absolute values. A corollary of Schmidt's theorem states that a field complete with respect to a discrete absolute value is not complete with respect to an absolute value inequivalent to the original. An application of this corollary is the following standard proof of the fact that the identity map is the only automorphism of the field Q_p of p-adic numbers: Let g be an automorphism of Q_p with p-adic absolute value $|\ |_p$, and define an absolute value $|\ |_g$ on Q_p by $|\alpha|_g = |g^{-1}(\alpha)|_p$ for all $\alpha \in Q_p$. Then Q_p is complete with respect to $|\ |_g$, $|\ |_p$ and $|\ |_g$ agree on Q, and $|\alpha|_p = |g(\alpha)|_g$ for all $\alpha \in Q_p$. Since $|\ |_p$ and $|\ |_g$ are equivalent, they agree on Q_p, and so $|\alpha|_p = |g(\alpha)|_p$ for all $\alpha \in Q_p$. Hence g is continuous and is, thus, the identity map.

It may be of interest to note that there is an alternative "elementary" proof that any automorphism of Q_p is continuous, based on the fact that such an automorphism must preserve the units of Q_p. That this is the case is an obvious consequence of the following algebraic characterization of the units.

THEOREM. *Let $\alpha \in Q_p$. Then $|\alpha|_p = 1$ if and only if α has an m-th root in Q_p for all positive integers m prime to $p(p-1)$.*

Proof. Sufficiency. Immediate from the fact that the range of $|\ |_p$ is the discrete set $\{0, p^n : n \in Z\}$.

Necessity. Since $|\alpha|_p = 1$, $\alpha = a_0 + a_1 p + a_2 p^2 + \cdots$, where $0 \leqq a_i < p$ and $a_0 \neq 0$. Consider the polynomial $f(x) = x^m - \alpha$. Since $(m, p-1) = 1$, it follows from a well-known fact about power congruences [3, p. 95] that there exists a natural number a such that $0 < a < p$ and $a^m \equiv a_0 (\text{mod}\, p)$. Hence $|f(a)|_p < 1$ and, since $(m, p) = 1$ and $0 < a < p$, $|f'(a)|_p = |ma^{m-1}|_p = 1$. Thus by Newton's method [2, p. 52] we may construct a sequence in Q_p which converges to a root of $f(x)$.

We remark that Ax and Kochen have proved a more general version of the preceding theorem as part of their study of formally p-adic fields [1, p. 633].

I wish to thank the referee for calling Schmidt's paper to my attention.

References

1. J. Ax and S. Kochen, Diophantine problems over local fields, II, Amer. J. Math., 87 (1965) 631–648.

2. G. Bachman, Introduction to p-adic Numbers and Valuation Theory, Academic Press, New York, 1964.

3. C. Long, Elementary Introduction to Number Theory, Heath, Boston, 1965.

4. F. K. Schmidt, Mehrfach perfekte Körper, Math. Ann., 108 (1933) 1–25.

DEPARTMENT OF MATHEMATICS, UNIVERSITY OF TENNESSEE, KNOXVILLE, TENNESSEE 37916.

A POINCARÉ TYPE COINCIDENCE THEOREM

SIMEON REICH

Let B denote the unit ball of a finite-dimensional Euclidean space E. According to Brouwer's fixed point theorem a continuous $g: B \to B$ has a fixed point. Let S denote the boundary of B. Recall that two functions f and g which map a set X into another set Y are said to have a coincidence if there exists a point $x \in X$ such that $f(x) = g(x)$. Schirmer [3] has established the following interesting coincidence theorem:

THEOREM 1. *Let f and g map B continuously into itself, and suppose that $f(S) \subset S$. If $f \mid S: S \to S$ is not nullhomotopic, then f and g have a coincidence.*

This proposition formally includes Brouwer's theorem because the identity map on S is not nullhomotopic. Of course, Brouwer's theorem is an immediate consequence of this (highly non-trivial) fact [1, p. 341].

Schirmer's proof is somewhat complicated. In this note we present a very simple proof of an extension of Theorem 1. It seems to be difficult to obtain this extension by adapting Schirmer's arguments. In the sequel, if $0 \neq x \in E$, then the point $x/|x|$, which belongs to S, will be denoted by $p(x)$.

THEOREM 2. *Let f and g map B continuously into E, and suppose that $f(S) \subset S$. If $f \mid S: S \to S$ is not nullhomotopic and $g(y) \neq mf(y)$ for all $y \in S$ and $m > 1$, then f and g have a coincidence.*