Logic and Proof II Carl Wagner

Logical Equivalence and Set-theoretic Identity

Logical Equivalence	Corresponding Set-theoretic Identity
1. De Morgan's laws	
$\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$	$(A \cap B)^c = A^c \cup B^c$
$\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$	$(A \cup B)^c = A^c \cap B^c$
2. Commutative laws	
$(P \wedge Q) \Leftrightarrow (Q \wedge P)$	$A \cap B = B \cap A$
$(P \vee Q) \Leftrightarrow (Q \vee P)$	$A \cup B = B \cup A$
$(P + Q) \Leftrightarrow (Q + P)$	$A \Delta B = B \Delta A$
3. Associative laws	
$P \wedge (Q \wedge R) \Leftrightarrow (P \wedge Q) \wedge R$	$A \cap (B \cap C) = (A \cap B) \cap C$
$P \vee (Q \vee R) \Leftrightarrow (P \vee Q) \vee R$	$A \cup (B \cup C) = (A \cup B) \cup C$
$P + (Q + R) \Leftrightarrow (P + Q) + R$	$A \Delta (B \Delta C) = (A \Delta B) \Delta C$
4. Idempotent laws	
$P \wedge P \Leftrightarrow P$	$A \cap A = A$
$P \vee P \Leftrightarrow P$	$A \cup A = A$
5. Distributive laws	
$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$P \wedge (Q + R) \Leftrightarrow (P \wedge Q) + (P \wedge R)$	$A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

6. Absorption laws

$P \wedge (P \vee Q) \Leftrightarrow P$	$A \cap (A \cup B) = A$
$P \vee (P \wedge Q) \Leftrightarrow P$	$A \cup (A \cap B) = A$

7. Double negation law

$\neg \neg P \Leftrightarrow P$	$(A^c)^c = A$

8. Tautology laws (U = the universal set)

$P \vee \neg P$ is a tautology	$A \cup A^c = U$
$P \wedge$ (a tautology) $\Leftrightarrow P$	$A \cap U = A$
$P \vee$ (a tautology) is a tautology	$A \cup U = U$
$P +$ (a tautology) $\Leftrightarrow \neg P$	$A \,\Delta\, U = A^c$
$\neg$(a tautology) is a contradiction	$U^c = \varnothing$

9. Contradiction laws (U = the universal set)

$P \wedge \neg P$ is a contradiction	$A \cap A^c = \varnothing$
$P \wedge$(a contradiction) is a contradiction	$A \cap \varnothing = \varnothing$
$P \vee$ (a contradiction) $\Leftrightarrow P$	$A \cup \varnothing = A$
$\neg P \rightarrow$ (a contradiction) $\Leftrightarrow P$	This equivalence, which justifies proof by contradiction, is just a variant of the preceding equivalence, since $\neg P \rightarrow Q \Leftrightarrow P \vee Q$.
$P +$ (a contradiction) $\Leftrightarrow P$	$A \,\Delta\, \varnothing = A$
$\neg$(a contradiction) is a tautology	$\varnothing^c = U$

10. Conditional equivalences

$P \rightarrow Q \Leftrightarrow \neg P \vee Q$

$\neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$

$P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R$

11. Biconditional equivalences

$P \leftrightarrow Q \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P) \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

$\neg(P \leftrightarrow Q) \Leftrightarrow P + Q$

12. Quantifier laws

$\neg \forall x\, P(x) \Leftrightarrow \exists x\, \neg P(x)$

$\neg \exists x\, P(x) \Leftrightarrow \forall x\, \neg P(x)$

$\forall x\, (P(x) \wedge Q(x)) \Leftrightarrow \forall x\, P(x) \wedge \forall x\, Q(x)$

$\forall x\, P(x) \vee \forall x\, Q(x) \Rightarrow \forall x\, (P(x) \vee Q(x))$

$\forall x\, (P(x) \vee Q(x))$ DOES <u>NOT</u> IMPLY $\forall x\, P(x) \vee \forall x\, Q(x)$!!!

$\exists x\, (P(x) \vee Q(x)) \Leftrightarrow \exists x\, P(x) \vee \exists x\, Q(x)$

$\exists x\, (P(x) \wedge Q(x)) \Rightarrow \exists x\, P(x) \wedge \exists x\, Q(x)$

$\exists x\, P(x) \wedge \exists x\, Q(x)$ DOES <u>NOT</u> IMPLY $\exists x\, (P(x) \wedge Q(x))$!!!