In 2010, NSF/DMS was externally reviewed by a Committee of Visitors (COV) to address its balance, priorities, and future directions, among other things. This COV review culminated in a report that found that DMS is underfunded and that, in spite of an overall budget increase, most of the increased funds went to interdisciplinary programs while funding of the core DMS programs stayed constant. The COV recommended that more money be directed to core areas.[10] In its response to the COV report, DMS pointed out that funding for core areas increased significantly from 2006 to 2007, with additional (but small) increases in the following 2 years.[11] DMS is faced with an innate conflict: As the primary funding unit charged with maintaining the health of the mathematical sciences, it is naturally driven to aid the expansions discussed in Chapter 3; yet it is also the largest of a very few sources whose mission includes supporting the foundations of the discipline, and thus it plays an essential role with respect to those foundations. As noted in Chapter 3, some mathematical scientists receive research support from other parts of NSF and from nonmath units in other federal funding agencies, but there are only anecdotal accounts of this. With limited data it is difficult to get a full picture of the totality of funding for the broader mathematical sciences community—the community that is an intellectually coherent superset of those researchers who sit in departments of mathematics or statistics—and to determine whether the funding is adequate and appropriately balanced. Nor can we say whether it is keeping pace with the expanding needs of this broader community. There are challenges inherent in supporting a broad, loosely knit community while maintaining its coherence, and the adequacy and balance of funding is a foremost concern. As noted in Chapter 3, funding of excellence wherever it is found should still be the top priority.

A VISION FOR 2025

Finding: Mathematical sciences work is becoming an increasingly integral and essential component of a growing array of areas of investigation in biology, medicine, social sciences, business, advanced design, climate, finance, advanced materials, and much more. This work involves the integration of mathematics, statistics, and computation in the broadest sense, and the interplay of these areas with areas of potential application; the mathematical sciences are best conceived of as including

[10] NSF/DMS Committee of Visitors, NSF/DMS, 2010, *Report of the 2010 Committee of Visitors*. Available at http://www.nsf.gov/attachments/117068/public/DMS_COV_2010_final_report.pdf.

[11] NSF, 2010, R*esponse to the Division of Mathematical Sciences Committee of Visitors Report*. Available at http://www.nsf.gov/mps/advisory/covdocs/DMSResponse_2010.pdf.

all these components. These activities are crucial to economic growth, national competitiveness, and national security. This finding has ramifications for both the nature and scale of funding of the mathematical sciences and for education in the mathematical sciences.

Conclusion 4-2: The mathematical sciences have an exciting opportunity to solidify their role as a linchpin of twenty-first century research and technology while maintaining the strength of the core, which is a vital element of the mathematical sciences ecosystem and essential to its future. The enterprise is qualitatively different from the one that prevailed during the latter half of the twentieth century, and a different model is emerging—one of a discipline with a much broader reach and greater potential impact. The community is achieving great success within this emerging model, as recounted in this report. But the value of the mathematical sciences to the overall science and engineering enterprise and to the nation would be heightened by increasing the number of mathematical scientists who share the following characteristics:

- **They are knowledgeable across a broad range of the discipline, beyond their own area(s) of expertise;**
- **They communicate well with researchers in other disciplines;**
- **They understand the role of the mathematical sciences in the wider world of science, engineering, medicine, defense, and business; and**
- **They have some experience with computation.**

It is by no means necessary or even desirable for all mathematical scientists to exhibit these characteristics, but the community should work toward increasing the fraction that does.

To move in these directions, the following will need attention:

- The culture within the mathematical sciences should evolve to encourage development of the characteristics listed in Conclusion 4-2.
- The education of future generations of mathematical scientists, and of all who take mathematical sciences coursework as part of their preparation for science, engineering, and teaching careers, should be reassessed in light of the emerging interplay between the mathematical sciences and many other disciplines.
- Institutions, for example, the funding mechanisms and reward systems—should be adjusted to enable cross-disciplinary careers when they are appropriate.
- Expectations and reward systems in academic mathematics and statistics departments should be adjusted so as to encourage a

broad view of the mathematical sciences and to reward high-quality work in any of its areas.

- Mechanisms should be created that help connect researchers outside the mathematical sciences with mathematical scientists who could be appropriate collaborators. Funding agencies and academic departments in the mathematical sciences could play a role in lowering the barriers between researchers and brokering such connections. For academic departments, joint seminars, cross-listing of courses, cross-disciplinary postdoctoral positions, collaboration with other departments in planning courses, and courtesy appointments are useful in moving this process forward.
- Mathematical scientists should be included more often on the panels that design and award interdisciplinary grant programs. Because so much of today's science and engineering builds on advances in the mathematical sciences, the success and even the validity of many projects depends on the early involvement of mathematical scientists.
- Funding for research in the mathematical sciences must keep pace with the opportunities.

While there are limits to the influence that it can have on the direction and character of research in the mathematical sciences and on the culture of the mathematical sciences community, the NSF can exercise leadership and serve as an enabler of positive developments. Successful examples include the flourishing Research Experiences for Undergraduates program and NSF's portfolio of mathematical science institutes. The NSF can, through funding opportunities, enhance the pace of change and facilitate bottom-up developments that capitalize on the energy of members of the community—examples include open calls for workforce proposals, grants to enable the development of new courses and curricula; grants that support interdisciplinary research and research between disciplines within the mathematical sciences, grants that enable individuals to acquire new expertise; and programs that make it easier for young people to acquire experience in industry and to acquire international experience.

The trends discussed in this chapter may appear quite disruptive to many core mathematicians, or even irrelevant. To address that possibility, the committee closes with a personal reflection by the study vice-chair, Mark Green, in Box 4-1, "Core Mathematicians and the Emerging Mathematical Landscape."

ematical sciences talent and sophistication to bear on ill-posed problems so as to make a contribution to the solution of these problems. This requires different skills from the ones that they trained for during their graduate student days, and it suggests that the training of graduate students in the mathematical sciences needs to be rethought given the changing landscape in which students may now work. At the least, mathematics and statistics departments should take steps to ensure that their graduate students have a broad and up-to-date understanding of the expansive reach of the mathematical sciences.

Recommendation 5-1: Mathematics and statistics departments, in concert with their university administrations, should engage in a deep rethinking of the different types of students they are attracting and wish to attract and must identify the top priorities for educating these students. This should be done for bachelor's, master's, and Ph.D.-level curricula. In some cases, this rethinking should be carried out in consultation with faculty from other relevant disciplines.

Recommendation 5-2: In order to motivate students and show the full value of the material, it is essential that educators explain to their K-12 and undergraduate students how the mathematical science topics they are teaching are used and the careers that make use of them. Modest steps in this direction could lead to greater success in attracting and retaining students in mathematical sciences courses. Graduate students should be taught about the uses of the mathematical sciences so that they can pass this information along to students when they become faculty members. Mathematical science professional societies and funding agencies should play a role in developing programs to give faculty members the tools to teach in this way.

The mathematical science community collectively does not do a good job in its interface with the general public or even with the broader scientific community, and improving this would contribute to the goal of enlarging the STEM pipeline. Internet tools such as blogs and video lectures offer new pathways for this outreach, which may be appealing to both practicing and retired mathematical scientists. There is a special need to improve the general level of understanding about uncertainty, which relies on an understanding of probability and statistics.

Recommendation 5-3: More professional mathematical scientists should become involved in explaining the nature of the mathematical sciences enterprise and its extraordinary impact on society. Academic departments should find ways to reward such work. Professional soci-

eties should expand existing efforts and work with funding entities to create an organizational structure whose goal is to publicize advances in the mathematical sciences.

Finally, the committee notes that the boom-and-bust cycles of the academic job market, especially for new Ph.D.s, result in a substantial loss of talent because they both discourage entry to research in the mathematical sciences and increase the likelihood of exit from it. The impact on core mathematical sciences, where the academic job market is central, is especially severe. Important workforce programs, such as NSF's former VIGRE program, are often dwarfed by these macroeconomic trends. Stabilizing these swings by expanding the availability of postdoctoral fellowships during downturns in the job market should be an important component of the nation's overall strategy to strengthen the mathematical sciences workforce and ensure continuity over long time horizons. NSF/DMS did just that during the recession of 2008-2009, and it would be ideal if a mechanism were in place to respond similarly during the next downturn in hiring.

It is because of the importance and centrality of the mathematical sciences, as detailed elsewhere in this report, that these educational issues are as important as they are. As a community, mathematical scientists have been handed an extraordinary opportunity to play a central role in educating researchers and professionals in many of the most exciting career and research areas of the twenty-first century. Taking advantage of this opportunity requires a certain amount of cultural flexibility and the development of educational partnerships with those in other disciplines. The benefits to the country and to the mathematical sciences profession would be enormous.

Appendix C provides additional basic data about employment and Ph.D. production in the mathematical sciences.

ATTRACTING MORE WOMEN AND UNDERREPRESENTED MINORITIES TO THE MATHEMATICAL SCIENCES

Concerns About the Current Demographics

The underrepresentation of women and ethnic minorities in mathematics has been a persistent problem for the field. Fifty years ago, the mathematical sciences community consisted almost exclusively of white males, and that segment of the population remains the dominant one from which the community attracts new members. This implies that talent in other sectors of the population is being underutilized, and as white males become a smaller fraction of the population, it is even more essential that the mathematical sciences attract and retain students from across the totality of the population. While there has been significant progress in the last