Melting and Freezing - Phase Change Processes

very familiar processes: melting ice cube, defrosting turkey, ...,
evaporation / condensation, ...

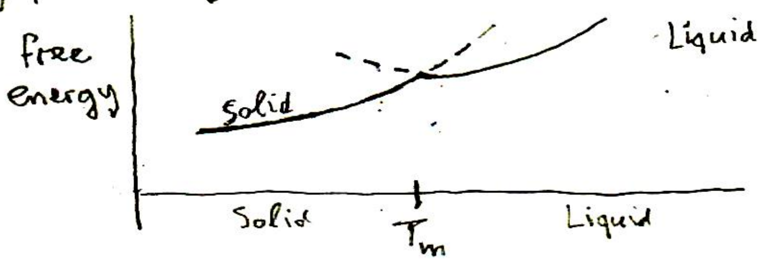
and not so familiar: crystal growth, laser ablation / annealing,
cryosurgery, Latent heat thermal energy storage,
...

Fundamental process in science, engineering, technology,
in materials science, environmental science,
geoscience, ...

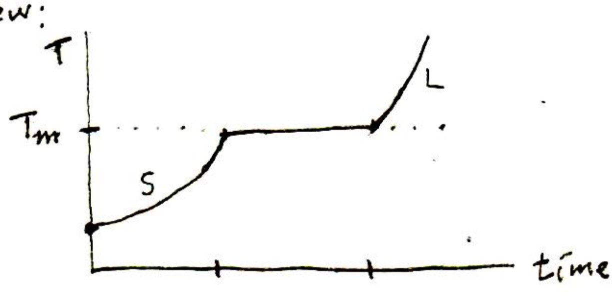
Physically belongs to Phase Transitions (of 1st order), physical chemistry
thermochemistry

Mathematically: moving boundary problem: in addition to $T(x,t)$
the location of solid-liquid interface also unknown.
So, region where PDE holds is itself unknown and
must be found. Makes problem nonlinear.

Why phase change occurs: to minimize free energy



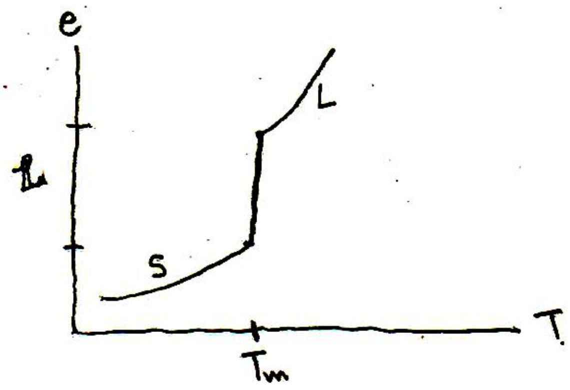
Thermocouple view:
melting case
(heating a solid)



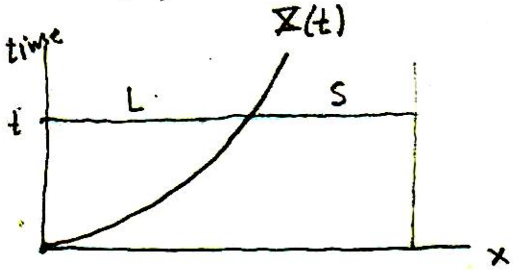
energy input
during transition
= Latent Heat
(heat of fusion)
= L [J/g]

enthalpy view: $e(T)$
(thermal energy)

$$e(T) = \begin{cases} e^S(T), & T < T_m \\ [0, \mathcal{L}], & T = T_m \\ e^L(T), & T > T_m \end{cases}$$



PDE view: find $T(x,t)$ and $X(t)$ = location of melt front



<u>Quantities</u>: $T(x,t)$ = temperature

T_m = melting temp.

$\vec{q} = -k \nabla T$ = heat flux (Fourier Law)

$e(T)$ = enthalpy (per gram), $E = \rho e$ per vol.

$\mathcal{L}$ = latent heat (per gram) = heat of fusion = $\Delta h(T_m)$

EoS $\quad \frac{de}{dT} = c_p$ = specific heat $\left[\frac{J}{gK}\right]$

Parameters: c_p^S, c_p^L, k^S, k^L, $\rho^S = \rho^L =: \rho$, $\mathcal{L}$

phases: λ = Liquid fraction
$1-\lambda$ = Solid fraction
super cooling: Liquid with $T < T_m$, super heating: Solid with $T > T_m$

heat transfer processes: <u>heat conduction</u>, convection, radiation