FV scheme for conservation law: $u_t + \mathcal{F}_x = S$ (in 1D)

$x \in \Omega = (a, b),\ t > 0$

$u(x,t)$: conserved quantity per unit volume at x, at time t

$\mathcal{F}(x,t)$: flux of u: amount crossing a unit area per unit time

$S(x,t)$: source of u: amount generated (or lost) per unit volume per unit time

Discetize space $[a,b]$ into M control volumes V_i, $i=1:M$

" time $[0, t_{end}]$ into N_{max} timesteps t_n, $n = 0 : N_{max}$

integrating the PDE over each V_i and over $[t_n, t_{n+1}]$ ... details later...

explicit FV scheme: $U_i^{n+1} = U_i^n + \dfrac{\Delta t}{\Delta x_i}\left[F^n_{i-\frac{1}{2}} - F^n_{i+\frac{1}{2}} \right] + \Delta t\, S_i^n$

$i = 1:M, \quad n = 0, 1, 2, \dots, N_{max}$

where

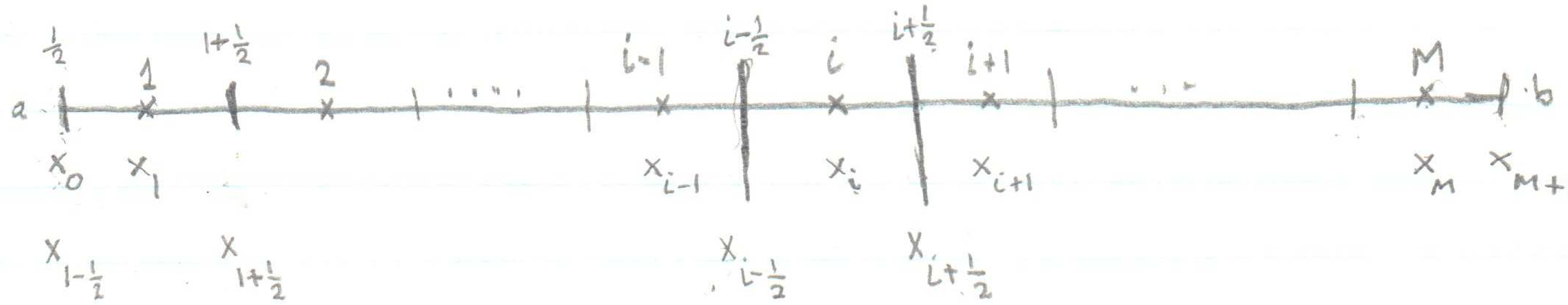
U_i^n = mean value of u over V_i at time t_n $= \dfrac{1}{V_i}\displaystyle\int_{V_i} u(x, t_n)\,dx \approx u(x_i, t_n)$

$F^n_{i-\frac{1}{2}}$ = mean flux during $[t_n, t_{n+1}]$ across face $x_{i-\frac{1}{2}}$ $= \dfrac{1}{\Delta t}\displaystyle\int_{t_n}^{t_{n+1}} F(x_{i-\frac{1}{2}}, t)\,dt$

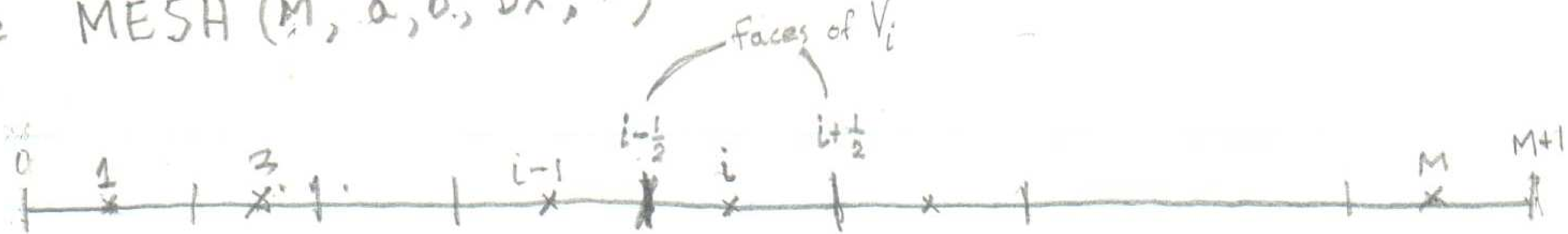
S_i^n = mean source over V_i and timestep $= \dfrac{1}{\Delta t}\dfrac{1}{V_i}\displaystyle\int_{t_n}^{t_{n+1}}\int_{V_i} S(x,t)\,dx\,dt$

$\Delta x_i = x_{i+\frac{1}{2}} - x_{i-\frac{1}{2}}$ = volume of $V_i = [x_{i-\frac{1}{2}}, x_{i+\frac{1}{2}}]$

$\Delta t = t_{n+1} - t_n$ = timestep



arrays: $X(0:M+1)$, $U(0:M+1)$, $F(1:M+1)$ no time-index routine MESH (M, a, b, Dx, x)



nodes array $x(0:M+1)$

simplest is uniform grid (of equispaced nodes and faces)

$$\Delta x = \frac{b-a}{M}$$

$$x(0) = a; \quad x(1) = x(0) + \frac{\Delta x}{2};$$

for $i = 2:M$

$$x(i) = x(1) + (i-1)\cdot\Delta x$$

$$x(M+1) = b$$

(check: $x(M) = a + \frac{\Delta x}{2} + (M-1)\Delta x$
$= \cancel{a} + \frac{\Delta x}{2} + b - \cancel{a} - \Delta x$
$= b - \frac{\Delta x}{2}$ ✓)