

Math 113 – Final Exam

14 December 2005

Name: _____

Directions: Answer every question. Show all work and justify your answers. Each question is worth five points.

1. Calculate the monthly loan payment in each of the three following situations.
 - (a) You buy a 2006 Hummer H1 and finance \$140,000 at an interest rate of 8% for five years.

 - (b) You buy a house in Knoxville and finance \$140,000 at an interest rate of 5.8% for 30 years.

 - (c) You go to Harvard Medical School and finance \$140,000 at an interest rate of 4.7% for 15 years.

2.
 - (a) List the first few Fibonacci numbers.

 - (b) What is the rule for producing the next number on the list?

3. (a) You make a single payment of \$480 into a bank account paying an interest rate of 4.2%. How much will be in the account after 4 years?
- (b) You put \$20 a month into a bank account paying an interest rate of 4.2%. How much will be in the account after 4 years?
4. List five of the problem-solving strategies that can be used when approaching an unfamiliar problem.
5. How would you determine the probability that it will snow in Knoxville on Christmas?
6. Determine whether each number is **rational** or **irrational**.
- (a) 2.37337333733337333337...
- (b) $\sqrt{16}$
- (c) $\frac{1}{53}$
- (d) $\sqrt{2}$
- (e) 1.234545454545454545...

7. You flip a penny, a nickel, and a dime.

(a) List the equally likely outcomes.

(b) What is the probability that the dime and the penny show different sides (one H the other T)?

(c) What is the probability of getting all Heads?

8. Kermit tells you that 48112959837082048697 is the largest prime number. (That number really is prime! I didn't just make it up!) How would you prove to Kermit that he is wrong?

9. **Prove or Disprove:** You can draw a connected planar graph with more faces than edges.

To **prove** this, draw such a graph. To **disprove** this, explain why it is impossible.

10. For each of the following pairings, determine whether or not it is a one-to-one correspondence.

(a) The pairing between the set $\{1, 2\}$ and the set $\{a, b, c\}$ given by

$$1 \leftrightarrow a$$

$$2 \leftrightarrow b$$

$$1 \leftrightarrow c$$

(b) The pairing between the set $\{1, 2, 3\}$ and the set $\{4, 5, 7\}$ given by

$$1 \leftrightarrow 4$$

$$2 \leftrightarrow 5$$

$$3 \leftrightarrow 7$$

(c) The pairing that matches people in the United States with phone numbers in service by matching each person with his or her phone number.

(d) The pairing that matches each real number with a natural number by matching each real number with the part to the left of the decimal point.

(e) The pairing that matches each positive real number with a negative real number by matching each number x with $-x$.

11. Sketch a representation of a 4-dimensional cube.

15. Give a real-world use of each of the following:

(a) Pythagorean Theorem

(b) Symmetry

(c) prime numbers

(d) probability

(e) Golden Ratio

16. You have a Golden Rectangle and the length of one side is given. Find the length of the other side.

(a) The shorter side is 4 meters long.

(b) The longer side is 16 meters long.

17. Explain how to make a Möbius Strip.

18. State the Pigeonhole Principle.

19. You are using the Art Gallery Theorem to determine the number of cameras necessary to guard a gallery with eight vertices.

(a) What is the most cameras you could possibly need?

(b) What is the fewest cameras might need?

(c) Sketch an 8-vertex gallery that requires this fewest number of cameras.

20. (a) **True or False (why or why not):** There are different sizes of infinity.

(b) **True or False (why or why not):** If you try to pair the natural numbers with the real numbers, every possible pairing that you attempt will have real numbers leftover.