

Math 113: Final Exam

Name: _____

Friday, May 1, 2009

Directions: Answer every question. Show appropriate work.

1. A financial advisor on AM talk radio suggests putting 15% of your monthly income in a retirement savings account. Consider a person who earns \$3000 a month.
 - (a) How much should this person save for retirement each month?

 - (b) If this person puts the money in an investment that pays 6%, how much will be in the account after 40 years?

2. Someone who can't quite make ends meet keeps charging things on a credit card. It started out as just a little bit here and there for essentials like groceries, but over time it has grown into \$8000 in credit card debt! The credit card charges 22% interest. Suppose this person gets fed up with being in debt, decides to get on a written budget, stops charging anything else on the card, and vows to pay off the credit card debt in four years.
 - (a) Calculate the monthly payment to pay off the \$8000 debt at 22% interest in 4 years.

 - (b) What is the total value of all 48 monthly payments? How much of this amount is interest paid to the credit card company?

6. What is wrong with the following “proof” of the Pythagorean Theorem?

The Pythagorean Theorem says that $a^2 + b^2 = c^2$. If you let $a = 5$ and $b = 12$ and $c = 13$, you get $5^2 + 12^2 = 13^2$, which simplifies to $25 + 144 = 169$. Since both sides equal 169, the Pythagorean Theorem must be true.

7. What is wrong with the following “disproof” of the Pythagorean Theorem?

The Pythagorean Theorem is wrong! It says that $a^2 + b^2 = c^2$. However, if you let $a = 2$ and $b = 8$ and $c = 10$, you get $2^2 + 8^2 = 10^2$, which simplifies to $4 + 64 = 100$. This isn't true because $68 \neq 100$. Therefore the Pythagorean Theorem is **false**.

8. What are the Fibonacci Numbers? What is the rule to produce the list of numbers?

9. What is a Golden Rectangle? If I gave you a rectangle, explain how you would decide if it was a Golden Rectangle.

10. The Art Department at the University of Tennessee offers concentrations in two-dimensional arts, three-dimensional arts, and four-dimensional arts. They break the curriculum down as follows:

2D: Drawing, painting, printmaking, and photography.

3D: Ceramics and sculpture.

4D: Film, video, and performance.

Do these categories make sense from a mathematical definition of dimension? Why or why not?

11. What is wrong with the following “proof” that $\sqrt{2}$ is irrational?

$\sqrt{2} = 1.414213562$. Since this doesn't repeat, it means that $\sqrt{2}$ is irrational.

12. What is wrong with the following “proof” that there are infinitely many prime numbers?

Numbers keep going on and on, so therefore there are infinitely many prime numbers.

13. Calculate:

(a) $144 + 439 \pmod{10}$

(b) $17^{1000} + 18^{1000} \pmod{17}$

(c) $3784 \pmod{3}$

14. List the names of the Platonic Solids:

4 faces:

6 faces:

8 faces:

12 faces:

20 faces:

15. An art gallery is shaped like a polygonal simple closed curve. It has 125 vertices. What does the Art Gallery Theorem say about the number of cameras necessary to watch the entire gallery? Be as specific as you can.

16. What is the role of prime numbers in RSA encryption? What mathematical principle is responsible for the security of this code?

17. What is a fractal? How can they be used in the real world?
18. A connected planar graph has 57 vertices and divides the plane into 80 regions. How many edges does it have?
19. Convert to a fraction (**show all work**): $0.853853853853853\dots$
20. List anything else you've learned in this class that hasn't appeared on this exam.