

4. Dover borrowed \$25,000 from the bank at a rate of 7.2% and will be paying this loan off for the next 10 years, making monthly payments of \$292.85.

(a) How many payments will Dover make?

(b) What is the total value of all the payments?

(c) How much goes to interest?

(d) Dover tells you, “The bank says that I can lower my monthly payments to only \$230.33 a month by spreading the loan out over 15 years, but they’d have to raise the interest rate to 7.4%.” What do you advise? You do not need to do any computations, just back up your opinion with general principles from financial math.

5. If I asked you to fill a container with approximately 1,000,000 grains of rice, how would you do it?

6. (a) Are the sets $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ and $\{1, 2, 3, 4, \dots\}$ equally numerous? (These sets are also known as the *integers* and the *natural numbers*.) Why or why not?

(b) Does the set of rational numbers have the same cardinality as the set of natural numbers? Very briefly explain your answer (you don't need to give a complete proof).

7. Cheyenne wants to have a million dollars in her account 40 years from now. She will make monthly payments into the account and expects to earn 4% interest. How much does she need to put into the account each month?

8. (a) Give a precise statement of the Art Gallery Theorem.

(b) Explain what it means in your own words.

9. (a) Find a list of seven numbers where the mean is larger than the median.

(b) Find a list of seven numbers where the median is larger than the mean.

10. In our proof of the number of Platonic Solids, we defined the quantities s and c , where s stood for the number of sides on a face and c stood for the number of edges leaving each vertex. We ended up with the three inequalities $s \geq 3$, $c \geq 3$, and $\frac{2}{c} + \frac{2}{s} > 1$. What are the allowable pairs of values of s and c ?

11. (a) List the equally likely outcomes of flipping a penny, a nickel, and a dime.

(b) What is the probability of getting at least one heads?

(c) If you know that the dime is heads, what is the probability that you got exactly two heads?

12. (a) What is the remainder when $(1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times 17 \times 18 \times 19 \times 20 \times 21 \times 22 \times 23 \times 24 \times 25 \times 26 \times 27 \times 28 \times 29 \times 30 \times 31 \times 33 \times 34 \times 35 \times 36 \times 37 \times 38 \times 39 \times 40 \times 41 \times 43 \times 44 \times 45 \times 46 \times 47 \times 48 \times 49 \times 50) + 1$ is divided by 7?

(b) Is 247 a prime number? How do you know?

13. (a) What does it mean for two objects to be *equivalent by distortion*?
- (b) Are \otimes and θ equivalent by distortion? Why or why not?
14. (a) Austin has drawn a right triangle. The lengths of the legs are 6 and 8. Use the Pythagorean Theorem to calculate the length of the hypotenuse.
- (b) Jackson has drawn a triangle whose sides have lengths 5.6, 7.1, and 9.8. Use the Pythagorean Theorem to determine whether or not this is a right triangle.
15. (a) Who were the Pythagoreans?
- (b) What bothered them about $\sqrt{2}$?

16. (a) Define *rigid symmetry*.

(b) Explain (in your own words) what it means when we say: The Pinwheel Pattern has a symmetry of scale, but it has no rigid symmetries.

17. (a) Give a completely accurate definition of *one-to-one correspondence*.

(b) When do two collections of objects have the same *cardinality*?

18. (a) Without referring to decimal expansions, give a definition of the *rational numbers*.
- (b) Is the quantity $3\sqrt{5}/9\sqrt{5}$ rational number? Why or why not?
- (c) Give an example of a rational number which is also a natural number.
19. The Hilbert Hotel has an infinite number of rooms — one for each natural number. The hotel is full. An infinite number of people arrive, wanting to stay at the hotel. Is there a way for the manager to give each person his own room (without kicking anyone out)? How can it be done or why is it impossible? (There is more than one right answer.)
20. At the hardware store, you bought 17 items (some screws and some nails) costing a total of 72 cents. A screw costs one cent more than a nail. How many screws did you buy? How many nails? How much did they cost each?

Bonus: What are some of the life lessons that we hope you learned in this course?

The following abbreviations will be used in formulas.

P	Principal – the original amount invested or borrowed
A	Account balance.
r	interest rate (written as a decimal)
Y	number of years
PMT	regular payment amount
n	number of payments per year (n is 12 for monthly payments)

If you invest $\$P$ at an interest rate of $r\%$ and leave the money in the account for Y years (untouched — making no deposits to or withdrawals from the account), the balance is given by the formula: $A = P(1 + r)^Y$. Similarly, if you borrow $\$P$ at an interest rate of $r\%$ and make no payments for Y years, the balance that you owe is given by the same formula: $A = P(1 + r)^Y$.

If you have a principal of $\$P$ invested at an interest rate of $r\%$, the interest that accumulates in one year is $P \times r$.

If you have a savings account which is earning $r\%$ interest and you deposit $\$PMT$ to the account n times a year and continue doing this for Y years, the amount of money in the account is given by the formula:

$$A = PMT \times \frac{\left[\left(1 + \frac{r}{n}\right)^{(nY)} - 1 \right]}{\left(\frac{r}{n}\right)}$$

If you borrow $\$P$ at an interest rate of $r\%$ and pay it back by making n equal payments a year for Y years, the size of the payment is given by the formula:

$$PMT = \frac{P \times \left(\frac{r}{n}\right)}{\left[1 - \left(1 + \frac{r}{n}\right)^{(-nY)}\right]}$$