

Name: _____

MATH 110 – FINAL EXAM
December 8, 2004
Dr. Szczepański. Version B.

Directions: There are twenty questions on this exam. Answer every question. Show all work and justify your answers. Each question is worth five points.

1. (a) Give a completely accurate definition of *one-to-one correspondence*.

(b) When do two collections of objects have the same *cardinality*?

2. (a) Without referring to decimal expansions, give a definition of the *rational numbers*.

(b) Is the quantity $3\sqrt{5}/9\sqrt{5}$ rational number? Why or why not?

3. The Hilbert Hotel has an infinite number of rooms — one for each natural number. The hotel is full. An infinite number of people arrive, wanting to stay at the hotel. Is there a way for the manager to give each person his own room (without kicking anyone out)? How can it be done or why is it impossible? (There is more than one right answer.)

4. At the hardware store, you bought 17 items (some screws and some nails) costing a total of 72 cents. A screw costs one cent more than a nail. How many screws did you buy? How many nails? How much did they cost each?

9. Dover borrowed \$25,000 from the bank at a rate of 7.2% and will be paying this loan off for the next 10 years, making monthly payments of \$292.85.
- (a) How many payments will Dover make?

 - (b) What is the total value of all the payments?

 - (c) How much goes to interest?
10. Dover (from the previous problem) tells you, “The bank says that I can lower my monthly payments to only \$230.33 a month by spreading the loan out over 15 years, but they’d have to raise the interest rate to 7.4%.” What do you advise? You do not need to do any computations, just back up your opinion with general principles from financial math.
11. Why did we study the Pinwheel Pattern? What notable features does it have (or lack)? Sketch a little bit of the pattern.
12. List the first few Fibonacci numbers and state the rule for finding the next number on the list.

13. (a) Give a precise statement of the Art Gallery Theorem.
- (b) Explain what it means in your own words.
14. List the five Platonic Solids. Draw a line from each solid's name to the name of its dual.
15. (a) What is the remainder when $(1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times 17 \times 18 \times 19 \times 20 \times 21 \times 22 \times 23 \times 24 \times 25 \times 26 \times 27 \times 28 \times 29 \times 30 \times 31 \times 32 \times 33 \times 34 \times 35 \times 36 \times 37 \times 38 \times 39 \times 40 \times 41 \times 43 \times 44 \times 45 \times 46 \times 47 \times 48 \times 49 \times 50) + 1$ is divided by 7?
- (b) Is 247 a prime number? How do you know?
16. (a) Who were the Pythagoreans? [Hint: "Mathematicians" is not good enough.]
- (b) What bothered them about $\sqrt{2}$?

17. (a) List the equally likely outcomes of flipping a penny and a dime.
- (b) What is the probability of getting at least one heads?
- (c) If you know that the dime is heads, what is the probability that you got exactly two heads?
18. (a) What does it mean for two objects to be *equivalent by distortion*?
- (b) Are ♡ and ◇ equivalent by distortion? Why or why not?
19. Austin has drawn a right triangle. One of the sides is 5 inches long, and another side is 10 inches long. What is the length of the third side? Find all possible solutions.
20. (a) What is the numeric value of the Golden Ratio?
- (b) List one reason why we studied the Golden Ratio.