1. [2 points each] For each pair of sets, determine whether they have the same cardinality or different cardinalities. You do not need to justify your answers.

(a) \{1, 2, 3, 4, 5, \ldots \} and \{2, 3, 4, 5, \ldots \}

(b) \{1, 2, 3, 4\} and \{9, 16, 17, 1004\}

(c) The set of natural numbers and the set of rational numbers.

(d) The set of natural numbers and the set of real numbers.

(e) \{1, 2, 3, 4, 5, \ldots \} and \{1, 2, 3, 4, 5, 6\}

2. [2 points each] Determine whether each of the following numbers are rational or irrational. You do not need to justify your answers.

(a) \frac{17}{49}

(b) \sqrt{16}

(c) 1.23456789101112131415\ldots

(d) \frac{3\sqrt{5}}{8\sqrt{5}}

(e) \sqrt{3}
3. [2 points each] True or False: Determine whether each statement is true or false. Give your answer as the entire word “true” or the entire word “false.” No credit for just T or F. You do not need to justify your answers.

(a) There are different sizes of infinity.

(b) \( \sqrt{2} = \frac{114243}{80782} \)

(c) There is a one-to-one correspondence between any two infinite sets.

(d) Every decimal number can be written as a fraction.

(e) The decimal expansion of \( \frac{5}{19} \) either terminates or is eventually periodic.

4. Using the method we learned in class, convert the decimal 12.34565656565656 \ldots into a fraction.
5. (a) Find a rational number between 3.1 and $\pi$.

(b) Find an irrational number between 3.1 and $\pi$.

6. Explain how you would determine whether or not there is a one-to-one correspondence between desks in this classroom and chairs in this classroom.

7. Find a one-to-one correspondence from the set of positive real numbers to the set of negative real numbers or explain why no such pairing exists. [Hint: Is there an obvious positive number that you would pair $-0.12345$ with?]

8. Dover says that he has found a one-to-one correspondence between the set of natural numbers and the set of real numbers. His pairing begins like this:

1 $\leftrightarrow$ 1.23456789101112 . . .
2 $\leftrightarrow$ 9.87654321012345 . . .
3 $\leftrightarrow$ 7.2322322322223 . . .
4 $\leftrightarrow$ 0.76567876545678 . . .
5 $\leftrightarrow$ 0.99999999999999 . . .

(a) Use Cantor’s diagonalization argument to come up with the first few decimal places of a number that you are absolutely certain is not on Dover’s list.

(b) What’s the importance of the existence of this number?
9. Prove that $\sqrt{17}$ is an irrational number.