

Math 113: Exam 2

Friday, February 23, 2007

Name: _____

Directions: Answer every question. Show appropriate work. Problems are five points each unless otherwise indicated.

1. [2 points each] For each pair of sets, determine whether they have the **same** cardinality or **different** cardinalities. You do not need to justify your answers.
 - (a) $\{1, 2, 3, 4, 5, \dots\}$ and $\{2, 3, 4, 5, \dots\}$
 - (b) $\{1, 2, 3, 4\}$ and $\{9, 16, 17, 1004\}$
 - (c) The set of natural numbers and the set of rational numbers.
 - (d) The set of natural numbers and the set of real numbers.
 - (e) $\{1, 2, 3, 4, 5, \dots\}$ and $\{1, 2, 3, 4, 5, 6\}$

2. [2 points each] Determine whether each of the following numbers are rational or irrational. You do not need to justify your answers.
 - (a) $\frac{17}{49}$
 - (b) $\sqrt{16}$
 - (c) 1.23456789101112131415...
 - (d) $\frac{3\sqrt{5}}{8\sqrt{5}}$
 - (e) $\sqrt{3}$

3. [2 points each] **True or False:** Determine whether each statement is true or false. Give your answer as the **entire word** "true" or the **entire word** "false." No credit for just T or F. You do not need to justify your answers.

(a) There are different sizes of infinity.

(b) $\sqrt{2} = \frac{114243}{80782}$

(c) There is a one-to-one correspondence between any two infinite sets.

(d) Every decimal number can be written as a fraction.

(e) The decimal expansion of $\frac{5}{19}$ either terminates or is eventually periodic.

4. Using the method we learned in class, convert the decimal $12.34565656565656 \dots$ into a fraction.

5. (a) Find a rational number between 3.1 and π .
- (b) Find an irrational number between 3.1 and π .
6. Explain how you would determine whether or not there is a one-to-one correspondence between **desks in this classroom** and **chairs in this classroom**.
7. Find a one-to-one correspondence from the set of **positive real numbers** to the set of **negative real numbers** or explain why no such pairing exists. [Hint: Is there an obvious positive number that you would pair -0.12345 with?]
8. Dover says that he has found a one-to-one correspondence between the set of natural numbers and the set of real numbers. His pairing begins like this:
- $$\begin{array}{l} 1 \leftrightarrow 1.23456789101112\dots \\ 2 \leftrightarrow 9.87654321012345\dots \\ 3 \leftrightarrow 7.23223222322223\dots \\ 4 \leftrightarrow 0.76567876545678\dots \\ 5 \leftrightarrow 0.99999999999999\dots \\ \vdots \end{array}$$
- (a) Use Cantor's diagonalization argument to come up with the first few decimal places of a number that you are absolutely certain is not on Dover's list.
- (b) What's the importance of the existence of this number?

9. Prove that $\sqrt{17}$ is an irrational number.