

3. Rewrite the decimal $5.94821212121212121\dots$ as a fraction. You do not need to reduce it to lowest terms.

4. Explain how (without counting) you would decide if Imelda Marcos has the same number of right shoes as left shoes.

5. Give precise and mathematically accurate definitions of:

(a) one-to-one correspondence

(b) rational numbers (do not refer to decimal expansions)

6. (a) Consider the set of people living in the United States and the set of phone numbers in service. Is the pairing that matches people to their phone numbers a one-to-one correspondence? Why or why not?
- (b) Consider the 52 cards in a standard deck of playing cards (no jokers). Is a pairing that matches the red cards to the black cards a one-to-one correspondence? Why or why not?
7. (a) Do the sets $\{1, 2, 3, 4, 5, 6, 7, \dots\}$ and $\{21, 22, 23, 24, 25, \dots\}$ have the same cardinality? Why or why not?
- (b) Do the sets $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ and $\{1, 2, 3, 4, \dots\}$ have the same cardinality? Why or why not?
8. The Hilbert Hotel has an infinite number of rooms — one for each natural number. The hotel is full. An infinite number of people arrive, wanting to stay at the hotel. Is there a way for the manager to give each person his own room (without kicking anyone out)? How can it be done or why is it impossible? (There is more than one right answer.)

9. Dover says that he has found a one-to-one correspondence between the set of natural numbers and the set of real numbers. His pairing begins like this:

$$\begin{aligned} 1 &\leftrightarrow 1.23456789101112\dots \\ 2 &\leftrightarrow 9.87654321012345\dots \\ 3 &\leftrightarrow 7.23223222322223\dots \\ 4 &\leftrightarrow 0.76567876545678\dots \\ 5 &\leftrightarrow 0.99999999999999\dots \\ &\vdots \end{aligned}$$

Use Cantor's diagonalization argument to come up with the first few decimal places of a number that you are absolutely certain is not on Dover's list.

10. Suppose that a baseball team with an infinite number of players (The Cardinals) wore uniforms labeled: 1, 2, 3, 4, 5, ... (one for each natural number). They are standing in line by uniform number. Each player has a penny, and you ask the players to flip their pennies at the same time. You ask them to shout out (in order) what they flipped (H for heads or T for tails). Consider the set of all possible outcomes of their flipping (all possible sequences of H and T). Does the set of possible outcomes have the same cardinality as the natural numbers or not? Justify your answer.