

Math 113: Exam 2

Friday, February 15, 2008

Name: _____

Directions: Answer every question. Show appropriate work. Problems are five points each unless otherwise indicated.

1. [2 points each] Check off all the boxes that apply for each number.

number	natural	integer	rational	irrational	real
-7	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$\sqrt{13}$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
102	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{2\sqrt{7}}{7\sqrt{7}}$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
3.141592653	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{2}{19}$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

2. [2 points each] For each pair of sets, determine whether they have the **same** cardinality or **different** cardinalities. You do not need to justify your answers.

(a) $\{1, 2, 3, 4, 5, \dots\}$ and $\{2, 3, 4, 5, \dots\}$

(b) $\{1, 2, 3, 4\}$ and $\{9, 16, 17, 1004\}$

(c) The set of natural numbers and the set of rational numbers.

(d) The set of natural numbers and the set of real numbers.

(e) $\{1, 2, 3, 4, 5, \dots\}$ and $\{1, 2, 3, 4, 5, 6\}$

3. Decide if the following equation is true or false and explain: $\frac{665857}{470832} = \sqrt{2}$

4. Who were the Pythagoreans? What did they do to the guy who revealed the secrets of irrational numbers?

5. Explain an efficient way to determine whether or not Imelda Marcos has the same number of left shoes as right shoes in her collection. (She claimed to have over 1000 pairs of shoes.)



6. Find a one-to-one correspondence from the set of **positive real numbers** to the set of **negative real numbers** or explain why no such pairing exists. [Hint: Is there an obvious positive number that you would pair -0.12345 with?]

7. Using methods we learned in class, either convert the decimal $3.455555555\dots$ into a fraction or explain why it's impossible.

8. When using Cantor's diagonalization argument, we started with a pairing that looked something like this:

$$\begin{array}{l} 1 \leftrightarrow 1.23456789101112\dots \\ 2 \leftrightarrow 9.87654321012345\dots \\ 3 \leftrightarrow 7.23223222322223\dots \\ 4 \leftrightarrow 0.76567876545678\dots \\ 5 \leftrightarrow 0.99999999999999\dots \\ \vdots \end{array}$$

- (a) Put boxes around the numbers on the diagonal, like we did in class.
- (b) Apply Cantor's method to come up with the number M , like we did in class.

- (c) What's the importance of the existence of this number?

9. Consider the following gameboards for the grid game. Decide whether or not Player 2 is correctly using the diagonal strategy. Justify your answer.

	X	O	O	X	X	O
	X	X	O	X	X	O
Player 1:	X	O	X	X	X	O
	X	O	O	O	X	O
	X	O	O	X	O	O
Player 2:	X	O	O	X	X	

10. Prove that $\sqrt{5}$ is an irrational number, using the proof that we did in class.