

Name: _____

MATH 110 – EXAM 2
21 February 2005
Dr. Szczepański. Version A.

Directions: There are ten questions on this exam. Answer every question. Show all work and justify your answers. Each question is worth five points.

1. For each number, determine whether it is **rational** or **irrational**. You do not need to justify your answer.
 - (a) $\sqrt{2} \times \sqrt{18}$
 - (b) 1.23456789101112131415...
 - (c) $\frac{12345}{67890}$
 - (d) $\frac{\sqrt{2}}{2}$
 - (e) $\sqrt{17}$
2. Rewrite the decimal 1.1792323232323... as a fraction. You do not need to reduce it to lowest terms.
3.
 - (a) **True or False:** We have two sets and have found a one-to-one correspondence between the elements of one set and the elements of the other set. This shows that these two sets have the same cardinality.
 - (b) **True or False:** We have two sets and have paired elements of the first set with elements of the second set in such a way that all the elements from the first set are paired with elements from the second set, but there are elements from the second set that were never paired. This shows that these two sets have different cardinality.
4. **True or False (why or why not):** The number $\sqrt{2}$ has an unending decimal expansion, but it might eventually repeat.
5. Consider the set of real numbers between 0 and 1 whose decimal expansions consist only of 3s and 7s. (Things like 0.777337373337... or 0.37773337377... and such.)
 - (a) Give an example of a rational number in this set. Why is your number rational?
 - (b) Give an example of an irrational number in this set. Why is your number irrational?
6. Prove that the cardinality of the positive real numbers is the same as the cardinality of the negative real numbers. [Hint: it is impossible to make a list of all the real numbers; do not try to make a list.]
7. Give precise and mathematically accurate definitions of:
 - (a) one-to-one correspondence

- (b) rational numbers (do not refer to decimal expansions)
8. (a) Do the sets $\{1, 2, 3, 4, 5, 6, 7, \dots\}$ and $\{21, 22, 23, 24, 25, \dots\}$ have the same cardinality? Why or why not?
- (b) Do the sets $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ and $\{1, 2, 3, 4, \dots\}$ have the same cardinality? Why or why not?
9. Dover says that he has found a one-to-one correspondence between the set of natural numbers and the set of real numbers. His pairing begins like this:

$$\begin{aligned} 1 &\leftrightarrow 1.23456789101112\dots \\ 2 &\leftrightarrow 9.87654321012345\dots \\ 3 &\leftrightarrow 7.23223222322223\dots \\ 4 &\leftrightarrow 0.76567876545678\dots \\ 5 &\leftrightarrow 0.99999999999999\dots \\ &\vdots \end{aligned}$$

Use Cantor's diagonalization argument to come up with the first few decimal places of a number that you are absolutely certain is not on Dover's list.

Bonus (2 points) True or False (why or why not): For each real number, there is only one way to write it as a decimal.