

Name: _____

MATH 113 – EXAM 2
30 September 2005

Directions: Answer every question. Show all work and justify your answers. Each question is worth five points.

1. For each number, determine whether it is **rational** or **irrational**. You do not need to justify your answer.

(a) $\sqrt{3} \times \sqrt{12}$

(b) $\frac{3}{51}$

(c) $\frac{1}{\sqrt[3]{2}}$

(d) 1.01001000100001000001...

(e) $\sqrt{17}$

2. Rewrite the decimal 10.2456456456456456... as a fraction. You do not need to reduce it to lowest terms. (Note: if you just use the Ans \triangleright Frac feature on your calculator, you will receive no credit.)

3. Two people are playing the dodgeball game that we talked about in class. Here are Player 1's first three moves:

X	X	O	O	X	O
O	O	X	O	X	X
O	X	O	X	O	X

- (a) Use the winning strategy to fill in the first three moves on Player 2's grid:

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- (b) What is the name of this strategy? How did it get its name?

4. The Hilbert Hotel has an infinite number of rooms — one for each natural number. The hotel is full. An infinite number of people arrive, wanting to stay at the hotel. Is there a way for the manager to give each person his own room (without kicking anyone out)? How can it be done or why is it impossible? (There is more than one right answer.)

5. (a) **True or False (why or why not):** There are different sizes of infinity.

- (b) **True or False (why or why not):** If you try to pair the natural numbers with the real numbers, every possible pairing that you attempt will have real numbers leftover.

10. Prove that the cardinality of the positive real numbers is the same as the cardinality of the negative real numbers. [Hint: it is impossible to make a list of all the real numbers; do not try to make a list.]
11. (a) Do the sets $\{1, 2, 3, 4, 5, 6, 7, \dots\}$ and $\{21, 22, 23, 24, 25, \dots\}$ have the same cardinality? Why or why not?
- (b) Do the sets $\{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$ and $\{1, 2, 3, 4, \dots\}$ have the same cardinality? Why or why not?
12. (a) Who were the Pythagoreans? [Hint: “mathematicians” is not good enough.]
- (b) What bothered them about $\sqrt{2}$? How did they react?