

FINANCIAL MATHEMATICS

One goal of this course is to for you to be able to solve problems that you may face in real life. Mathematics can be used to help make financial decisions.

The following abbreviations will be used in formulas.

P	Principal – the original amount invested or borrowed
A	Account balance.
r	interest rate (written as a decimal)
Y	number of years
PMT	regular payment amount
n	number of payments per year (n is 12 for monthly payments)

If you invest $\$P$ at an interest rate of $r\%$ and leave the money in the account for Y years (untouched — making no deposits to or withdrawals from the account), the balance is given by the formula: $A = P(1 + r)^Y$. Similarly, if you borrow $\$P$ at an interest rate of $r\%$ and make no payments for Y years, the balance that you owe is given by the same formula: $A = P(1 + r)^Y$.

If you have a principal of $\$P$ invested at an interest rate of $r\%$, the interest that accumulates in one year is $P \times r$.

If you have a savings account which is earning $r\%$ interest and you deposit $\$PMT$ to the account n times a year and continue doing this for Y years, the amount of money in the account is given by the formula:

$$A = PMT \times \frac{\left[\left(1 + \frac{r}{n}\right)^{(nY)} - 1 \right]}{\left(\frac{r}{n}\right)}$$

There are two ways to use this formula: You may be given the size of the payment and asked to calculate the amount of money in the account or you may be given the amount that you want to have in the account and be asked to calculate the size of the payment.

Amortization is the process of paying off a loan by making equal monthly payments. If you borrow $\$P$ at an interest rate of $r\%$ and pay it back by making n equal payments a year for Y years, the size of the payment is given by the formula:

$$PMT = \frac{P \times \left(\frac{r}{n}\right)}{\left[1 - \left(1 + \frac{r}{n}\right)^{(-nY)}\right]}$$

There are many online calculators which simplify these calculations. One can be found at:

<http://www.myfico.com/myfico/CreditCentral/Calculators.asp>

1. You borrow \$100,000 to buy a house. Your mortgage has an interest rate of 6.25%, and you will be making monthly payments for the next 30 years. How much is your monthly payment?
2. You deposit \$500 into a savings account paying 3% interest. How much money is in the account after 4 years?
3. How much would you have to deposit today into a savings account earning 2.5% interest if you wanted to have \$1000 in the account after 8 years?
4. You have \$600 invested at 1.3%. How much interest will you earn in one year?
5. Every month you are putting \$50 into an investment which is paying 5.25% interest. How much will be in the account after 10 years?
6. You have \$5000 in credit card debt at an interest rate of 18%. If you don't charge anything else to the credit card, how much do you need to pay per month in order to pay off the debt in 4 years?
7. You overhear someone bragging about how she doesn't have to work because she's living off the interest of her investments. If she's earning \$50,000 in interest every year and her investments are paying an interest rate of 6.4%, how large are her investments?
8. You are saving money to buy a new car 5 years from now. You expect that the car will cost \$23,000. You are going to make a deposit every month into an account earning 2.8% interest. How much do you need to put into the account each month in order to reach your goal?
9. When someone (usually a bank, a retirement plan, or an insurance company) owes you money and sends you regular payments, it's called an annuity. Annuity payments are calculated exactly like loan payments. The interest rate for an annuity is often very similar to the rate of inflation, and P is called the present value of the annuity. Compute the monthly payment you would receive from a 15-year annuity with a present value of \$1,000,000 with a rate of 1.8%.
10. You have just won \$6,000,000 in the lottery. The state has given you two options as to how you can receive your money. You can receive an annuity which pays \$25,000 a month for 20 years or you can receive a lump sum payment of \$5,000,000 right now. Assume an inflation rate of 1.9%.
 - (a) What is the total amount of money which you receive if you accept the monthly payments?
 - (b) To determine which is a better deal, compute the monthly payment for an annuity with a present value of \$5,000,000. Compare this payment to the \$25,000 a month which the lottery has offered you—which is a better deal?
 - (c) What if the rate was 1.7%?
11. You put \$1000 in a bank account which is paying 4.1% interest.
 - (a) How much interest would you earn in a year?

- (b) How much interest would you earn in six months (half a year)?
 - (c) How much interest would you earn in one month?
12. You are trying to save \$1,000,000 by making a single deposit to a savings account and letting interest accumulate for several years. How much do you need to deposit in the following situations?
- (a) The account is earning 2% interest and you leave the money in the account for 20 years.
 - (b) The account is earning 2% interest and you leave the money in the account for 30 years.
 - (c) The account is earning 2% interest and you leave the money in the account for 40 years.
 - (d) The account is earning 4% interest and you leave the money in the account for 20 years.
 - (e) The account is earning 4% interest and you leave the money in the account for 30 years.
 - (f) The account is earning 4% interest and you leave the money in the account for 40 years.
13. You are trying to save \$1,000,000 over the next 30 years by regularly placing the same amount of cash each month into a cardboard box in your closet. How much do you need to add to the box each month?
14. You are trying to save \$1,000,000 by making a deposit every month into an account. How much do you need to deposit each month in the following situations? How much of the million dollars comes from the interest that you've earned?
- (a) The account is earning 2% interest and you leave the money in the account for 30 years.
 - (b) The account is earning 2% interest and you leave the money in the account for 40 years.
 - (c) The account is earning 4% interest and you leave the money in the account for 30 years.
 - (d) The account is earning 4% interest and you leave the money in the account for 40 years.
15. You are saving \$100 a month for 30 years. How much will you have in each of the following situations?
- (a) You put the money in a cardboard box.
 - (b) You put the money in a savings account earning 1.9% interest.
 - (c) You invest the money in bonds earning 4% interest.
 - (d) You invest in a mutual fund which grows (on average) by 6.7% a year.
 - (e) You invest in stocks which grow (on average) by 9% a year.

16. For 5 years, Shannon puts \$250 a month into a company-sponsored retirement account which is paying 4% interest. After the 5 years are over, no more deposits are made to the account, and the money which has accumulated continues to earn 4% interest for 25 more years.
- (a) How much is in Shannon's account at the end of the 30 years?
 - (b) Shannon's coworker, Lee, wants to have the same amount of money after 30 years. Unfortunately, Lee is not as conscientious about saving. If Lee puts nothing in the account for the first 25 years, how much will need to be deposited each month for the remaining 5 years in order to reach the savings goal?
17. You want to live off your investments when you retire 40 years from now. For this problem assume that you can always find investments which earn 6% interest.
- (a) If you want your investments to provide you with an income of \$150,000 a year, how much will you need to have in your account when you retire?
 - (b) How much will you need to invest each month in order to achieve this savings goal?
18. You borrow \$150,000 to buy a house. Your mortgage has an interest rate of 6.4%, and you will be making monthly payments. In each of the following situations, find the monthly payment, the total value of all the payments made, and the amount which goes to interest.
- (a) You pay the money back over 30 years.
 - (b) You pay the money back over 20 years.
 - (c) You pay the money back over 15 years.
19. You borrow \$130,000 to buy a house. You will be making monthly payments for the next 30 years. In each of the following situations, find the monthly payment, the total value of all the payments made, and the amount which goes to interest.
- (a) Your loan has an interest rate of 5.25%.
 - (b) Your loan has an interest rate of 6.25%.
 - (c) Your loan has an interest rate of 7.25%.
 - (d) (Needs an amortization table.) For each of these interest rates, how long does it take for you to pay off half the principal?
20. You are borrowing \$175,000 to buy a house, and you will be making monthly payments for the next 30 years. You have a choice of two loans. The first loan has an upfront fee of 1% of the amount borrowed and charges an interest rate of 5.6%. The second loan has no fees, but it has an interest rate of 5.9%.
- (a) Compute the monthly payment for each loan.
 - (b) Which loan is a better deal if you plan to sell the house after 1 year?
 - (c) Is this still the case if you plan to keep the house for the full 30 years?

21. You are borrowing \$18,000 to buy a new car. You will be making monthly payments for the next 5 years, and your loan has an interest rate of 6.1%.
- (a) How much is the monthly payment for this loan?
 - (b) Instead of buying a car, you decide to make due by riding your bicycle and taking the bus for the next five years and to deposit, each month, the amount you would have paid as a car payment into a savings account earning 4% interest. How much is in the account at the end of five years?
22. You want to buy a \$21,000 car, and you have saved \$19,000. Because of your very good credit and a promotion at the dealership, you qualify for a 3 year auto loan at an interest rate of 0.9% or \$2000 cash back. Which is a better deal: Do you make a down payment of \$2100, finance the car, keep the rest of the money in a savings account earning 2%, and use money from the account to make the car payments. Or do you buy the car outright and own it debt-free?