Problem Solving

One goal of this course is to for you to be able to solve problems of a type that you have never seen before. You should learn how to start thinking about an unfamiliar problem, how to use several different strategies to reach an answer, and how to check your answer to see if it is reasonable.

George Polya suggests a four-step approach to solving problems:

1. Understand the problem.
2. Devise a plan.
3. Carry out the plan.
4. Look back and check your answer.

Some strategies that you might use when devising or carrying out the plan:

- Draw a picture.
- Build a model.
- Guess and check.
- Solve a similar but simpler problem.
- Make a table or chart.
- Find a pattern.
- Work backwards.
- Use technology.
- Break the problem up into smaller problems.
- Rephrase in terms of a problem you've already solved.
- Combine strategies.

Problems

1. You are organizing a tournament involving 15 teams. How many games will have to be played so that every team has played each other team once?

2. You have just landed a job working on a ranch. Your first assignment is to help dig a ditch 720 feet long which will be used to bring water to the ranch animals. You and two other people, Jack and Sarah, will be digging the ditch. The ranch foreman claims that if all of you keep digging steadily, the job will be completed in 3 days. But at the end of the first day, Jack hurts his back and can't continue the job. You notice that all three of you seem to have been progressing at about the same rate. How long will it take you and Sarah to finish the job?

3. The zoo is ordering birdseed. The keeper of the bird cages knows that two cockatoos will eat 2 pounds of seed every 2 weeks, three parrots will eat 3 pounds of seed every 3 weeks, and 4 macaws will eat 4 pounds of seed every 4 weeks. How much birdseed should the zoo order for 12 cockatoos, 12 parrots, and 12 macaws for 12 weeks?

4. If you draw 15 dots around a circle, how many lines can you draw connecting pairs of dots?

5. A shepherd has 200 sheep. All but 20 of them die. How many lived?

6. At a high school reunion, 100 alumni decided to play an old locker game that involved opening and closing the lockers numbered 1 to 100. Initially, all the lockers were closed. The first person opened all the lockers. The second person then started at the beginning and closed every second locker. The third person then proceeded to change the state of every third locker. That is, she closed the open ones and opened the closed ones. The fourth and remaining people followed in a like manner, each changing the state of the fourth, fifth, etc. lockers. When the last person was finished, which lockers were open?

7. You are selling lamps at a craft fair. You are required to collect 9.25% sales tax on all of your sales. If you want a lamp to cost $20 including the tax, what pre-tax price should you charge?

8. Three clubs were meeting on the same day at the same time: cheerleaders, pep club, and student council. At each meeting not all club members were present because some students belong to more than one club. Half of the student council members and all the cheerleaders belong to the pep club. Two students are members of all three clubs. If there are 24 student council members, 6 cheerleaders, and 40 pep club members, how many students belong only to the pep club?

9. A club with 150 members is having a ping pong tournament. When a member loses a game, the member is out of the tournament. There are no ties. How many games must be played in order to determine the champion?

10. If you have a 5-quart pail and a 9-quart pail (and no other containers), how can you go to a water source and bring back exactly 7 quarts of water?

11. A farmer looks out into the barnyard and sees the pigs and chickens. He says to his daughter, "I count 169 heads and 398 feet." How many pigs and how many chickens are there?

12. A restaurant offers five different toppings for its pizzas. How many ways can you select two toppings? three toppings?

13. A restaurant offers 15 different toppings for its pizzas. How many ways can you select two toppings?

14. My cat is older than your cat. Furthermore, twice my cat's age (in years) plus your cat's age in years equals seven. How old are our cats?

15. Three travelers meet. They sit down to eat. One traveler produces three loaves of bread; the second traveler produces four loaves. After the seven loaves are divided equally and eaten, the third traveler produces seven dimes and says, "Please divide these dimes fairly between the two of you." How much money should each of the first two travelers receive?

16. On the final day of his close-out sale, a merchant hastily disposed of two lamps at the bargain price of twelve dollars apiece. He estimated that he must have made some profit on the combined transactions since he made a 25 percent profit on one and only took a 20 percent loss on the other. How much of a profit (or loss) did he make?

17. You drive to Asheville at an average speed of 60 miles per hour. There is construction on the interstate and an accident slowing up traffic on your way back, and you average a speed of 30 miles an hour on your trip back to Knoxville. What was your average speed for the round trip? (Hint: it doesn't matter how far it is to Asheville.)

18. How many cubic inches of dirt are in a hole that is one foot deep, two feet wide, and six feet long?

19. At the cafeteria (open seven days a week, 365 days a year) the soup of the day repeats every four days. The dessert repeats every 13 days, and the entree repeats every seven days. How long does it take for an entire meal to repeat?

20. If a hundred chickens eat a hundred bushels of grain in a hundred days, how many bushels will ten chickens eat in ten days? And if, on the average, one and a half of these chickens lay an egg and a half in a day and a half, how many days will it take a chicken to lay one and a half dozen eggs?

21. You are working at an inn about an hour south of here. During the busy season, your inn is booked up with tourists, and your boss has decided to run a shuttle to the airport. Your boss wants a shuttle to leave the inn every 10 minutes. You have been put in charge of arranging this with the van company. The van

company tells you that it will take 52.5 minutes to drive from the inn to the airport and that you should allow 7.5 minutes for loading and unloading at each stop. How many vans will you need?

22. In a cross-country run, Sven placed exactly in the middle among all participants. Dan placed lower (i.e. did worse than Sven), in tenth place, and Lars placed sixteenth. How many runners took part in the race?

23. Maia bought two unusual sandglasses (hourglasses). One measures a nine-minute interval, and the other measures a thirteen-minute interval. A certain love potion needs to boil for exactly thirty minutes. Is it possible to measure such a time interval with these sandglasses under the additional stipulation that you turn over the glass(es) for the *first time* just as the potion starts to boil?

24. You have a strip of paper that is two-thirds of a meter long. However, you need a strip exactly half a meter long. Must you have to ruler to cut off such a length?

25. One hundred children in a school are counting their money. Each child has between 1 and 100 cents, and no child has the same amount as any other child. Is it possible to divide the children into two groups so that no child in either group will have twice as much money as any other child in the same group?

26. An experiment is performed in which each trial consists of tossing an ordinary six-sided die repeatedly and adding the numbers that come up; in each trial, as soon as the total *exceeds* 15, we stop tossing the die. For this experiment, what final total is expected to occur most often?

27. How many numbers are there between 1 and 1000 whose digits sum to 3?

28. It takes a blacksmith five minutes to put on a horseshoe. How long would it take for eight blacksmiths to shoe ten horses? A horse can not stand on two legs — it can only have one shoe put on at a time.

29. The shapes in Tetris are called "tetrominoes." Each tetromino is made up of four squares. Draw all of the possible tetris shapes (there are seven of them counting mirror reflections). Which of the Tetris shapes can be used to tile a 10×10 grid (with no gaps between the shapes)?

30. A fire fighter stood on the middle rung of a ladder, directing water into the burning building. As the smoke diminished, she stepped up three rungs and continued her work from that position. A sudden flare-up of flames forced her to descend five rungs. A few minutes later she climbed up seven rungs and worked there until the fire was out. Then she climbed the seven remaining rungs up to the top. How many rungs are on the ladder?