

1. One has an exact first order differential equation

$$M(x, y) dx + N(x, y) dy = 0$$

when it is possible to find a function in two variables  $F(x, y) = C$  whose total differential coincides with the given equation. That is,

$$dF = \underbrace{\frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy}_{\text{Total Differential of } F(x,y)} = M(x, y) dx + N(x, y) dy = 0.$$

But, for  $F(x, y) = C$  to have a total differential around a point  $(x_0, y_0)$  there must exist a rectangle  $R$  with  $(x_0, y_0)$  in the interior, where the partial derivatives of the function  $F(x, y) : \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial^2 F}{\partial y \partial x}$ , and,  $\frac{\partial^2 F}{\partial x \partial y}$  exist and are continuous for every point  $(x, y)$  in  $R$ . If that is so, then, the mixed partial derivatives are equal

$$\frac{\partial^2 F}{\partial x \partial y} = \frac{\partial^2 F}{\partial y \partial x}.$$

This fact is known sometimes as **Clairaut's Theorem** sometimes as **Euler's Theorem for mixed derivatives** and also as **Schwarz's Theorem**.

The goal of these homework is to investigate what was the input made by these mathematicians, and try to determine who deserves credit, and for what exactly, so, please, hit the books the internet, and if necessary ask your professors in mathematics and physics for more bibliographical references to answer the following questions.

1.1 When Euler established his result about the mixed partial derivatives, was he working with objects in the  $xy$ -plane? Please answer only, yes or not.

Answer:	<input type="text"/>
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1.2 When Clairaut established his result about the mixed partial derivatives, was he working with objects in the  $xyz$ -space? Please answer only, yes or not.

Answer:	<input type="text"/>
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1.3 When Hermann Schwarz established his result about the mixed partial derivatives, was he working with objects in the  $n$ -th dimensional space,  $R^n$ ? Please answer only, yes or not.

Answer:	<input type="text"/>
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2. Fill out the following table. Continue on the back of this page, do not attach extra pages, please.

Give the name and the year of the article where Euler published his result about the mixed partial derivatives.
Give the name and the year of the article where Clairaut published his result about the mixed partial derivatives.
Give the name and the year of the article where Schwarz published his result about the mixed partial derivatives.

Give a brief description of Euler's work at the time of his publication.

Give a brief description of Clairaut's work at the time of his publication.

Give a brief description of Schwarz's work at the time of his publication.