

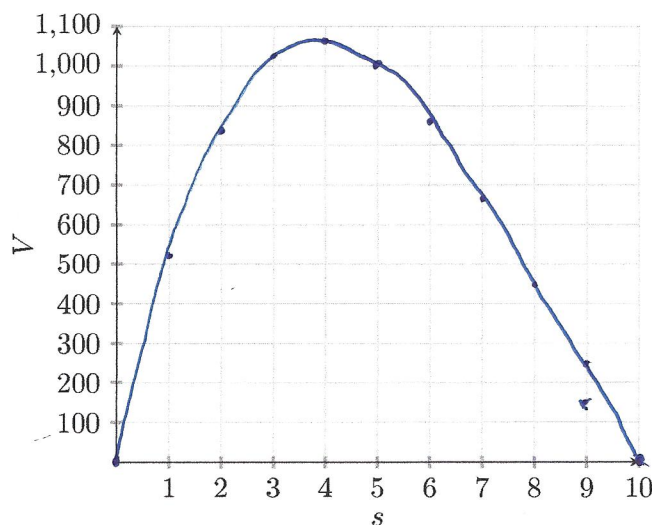
4.7 Applied Optimization

Example 4.7.1. Your group is given a piece of graph paper that was originally 20 units by 30 units. Four congruent squares have been cut from each corner. The size of the squares is different for each group. Fold the sides of your paper up to make a box with an open top.

Record the following (in graph paper units).

	$s=1$	$s=2$	$s=3$	$s=4$	$s=5$	$s=6$	$s=7$	$s=8$	$s=9$
Length of box:	28	26	24	22	20	18	16	14	12
Width of box:	18	16	14	12	10	8	6	4	2
Height of box:	1	2	3	4	5	6	7	8	9
Volume of box:	504	832	1008	1056	1000	864	672	448	216

Now let's plot a graph of the volumes, V , each group found based on the length of the side of the square, s , cut out from each corner.



Based on this graph, what is the approximate maximum volume?

$$\approx 1056 \text{ units}^3$$

Approximately what size square cut out of the corners results in this maximum volume?

$$\approx 4 \text{ units}$$

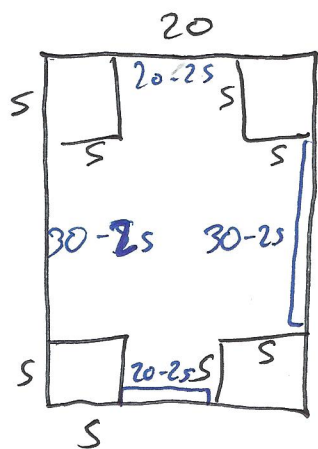
How could you use calculus to find the exact maximum volume?

If we had a function for Volume, we could take derivative and find CP and then determine where the maximum occurs.

Observe that the volume is determined by the side length s of the square cut from each corner (a quantity that we can control). We will now use calculus to figure out exactly what size square to cut from each corner in order to maximize the volume of the open-topped box.

Finding the maximum volume using calculus:

(a) Write a function for the volume (that will depend on s).



$$V = lwh$$

$$l = 30 - 2s$$

$$w = 20 - 2s$$

$$h = s$$

$$V = (30 - 2s)(20 - 2s)s$$

$$\Rightarrow V(s) = 600s - 100s^2 + 4s^3$$

(b) What is the domain of your function that fits the context of this problem?

$$0 \leq s \leq 10 \quad [0, 10]$$

(c) Find the maximum volume using calculus.

$$V'(s) = 12s^2 - 200s + 600$$

$$s = \frac{200 \pm \sqrt{200^2 - 4 \cdot 600 \cdot 12}}{2 \cdot 12}$$

$$\Rightarrow s \approx 12.7429 \rightarrow \text{not in domain}$$

$$s \approx 3.92375$$

$$V(0) = 0$$

$$V(3.92375) = 1056.31$$

$$V(10) = 0$$

$$\text{max Volume: } 1056.31 \text{ m}^3$$

The methods you have learned so far for finding extreme values have practical applications in many areas of life:

- A businessperson wants to minimize costs and maximize profits.
- A traveler wants to minimize transportation time.
- A migrating bird wants to maximize the distance it can travel without stopping, given the energy that can be stored as body fat.

In this section, we will apply your knowledge of calculating absolute minimums and absolute maximums to solve **optimization problems**.

Goal of an optimization problem: *Make a decision that is best possible for some quantity (maximize/minimize that quantity)*

The function that we are trying to maximize or minimize is called the objective function. Oftentimes, but not in every situation, our goal is restricted by something that we refer to as a **constraint**.

Examples:

- A businessperson wants to maximize his or her profits but is constrained by *market, money, profit, time, etc.*
- A traveler wants to minimize transportation time but is constrained by *traffic, speed limit, etc.*

Steps for Solving Optimization Problems:

picture 1. Draw a picture and choose your variables.

dependent variable: *the one(s) you're maximizing or minimizing*

independent variable(s): *the one(s) you control*

eqn 2. Create your objective function by relating your variables.

solve for variable 3. If the **objective function** is written as a function of **more than one variable**, then you will need to use a **constraint equation**. Solve your constraint equation for one variable and plug that in to your objective function so that it becomes a function of only one variable.

domain 4. Find the domain of your objective function that fits the context of the problem.

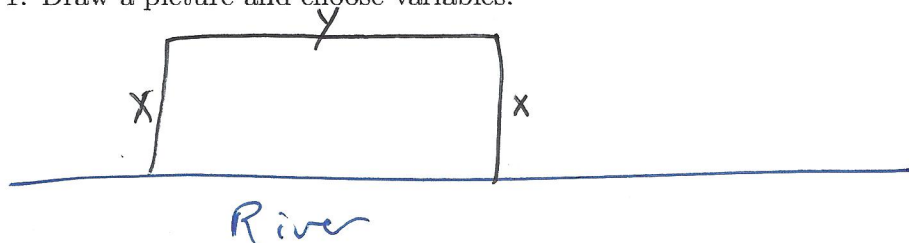
Use calculus tools to find an extreme value of the function on the domain:

CP 5. Find derivative and critical points (Do **not** stop at this step! You must verify whether or not the critical point gives a global max or min.)

max/min 6. Find global min or max

Example 4.7.2. A farmer has 2400 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

1. Draw a picture and choose variables.



2. Create your objective function.

$$A = xy$$

3. Get your objective function into one variable using a constraint equation.

$$2x + y = 2400 \text{ (const. eqn)}$$

$$\Rightarrow y = 2400 - 2x$$

$$\Rightarrow A = xy = (x)(2400 - 2x) = 2400x - 2x^2$$

4. Find the domain of your objective function.

$$\begin{aligned} 0 &\leq y \\ 0 &\leq x \leq 1200 \end{aligned} \Rightarrow y = 2400 - 2x \geq 0 \Rightarrow 2400 \geq 2x \Rightarrow 1200 \geq x$$

Use calculus to find the maximum area.

$$A'(x) = 2400 - 4x$$

$$A'(x) = 0 \Leftrightarrow x = 600$$

$$A(0) = 0$$

$$A(600) = 720,000 \text{ ft}^2$$

$$A(1200) = 0$$

Dimensions of field
w/ largest area:

$$x = 600 \text{ ft}, y = 2400 - 2 \cdot 600 = 1200 \text{ ft}$$