

4.5 L'Hôpital's Rule

Theorem 4.5.1 (L'Hôpital's Rule). If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ has indeterminate form " $\frac{0}{0}$ " or " $\frac{\infty}{\infty}$ ", then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

This also holds for $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$.

$$\text{Example 4.5.1. } \lim_{x \rightarrow 0} \frac{\sin x}{x} \stackrel{\text{L'Hop}}{=} \lim_{x \rightarrow 0} \frac{\cos x}{1} = 1$$

$$\text{Example 4.5.2. } \lim_{x \rightarrow \infty} \frac{2x+1}{7x^2+51} \stackrel{\text{L'Hop}}{=} \lim_{x \rightarrow \infty} \frac{2}{14x} = 0$$

WARNINGS:

- Don't mix up Quotient Rule + L'Hôpital's Rule!
- Don't use L'Hôpital's Rule if you don't have " $\frac{0}{0}$ " or " $\frac{\infty}{\infty}$ "

$$\text{Example 4.5.3. } \lim_{x \rightarrow \infty} xe^{-x}$$

$$\text{Example 4.5.4. } \lim_{x \rightarrow 1} \frac{x^2 - 1}{3x - 8}$$