Consider the functions $f(x) = e^x$ and $g(x) = x^{1,000,000}$. As $x \to \infty$, which of the following is true?

(a) f grows faster than g

(b) g grows faster than f

(c) We cannot determine which grows faster

(d) They grow at the same rate like all exponentials

x^2 grows faster than x

$$\lim_{x\to\infty} \frac{x^2}{x} = \lim_{x\to\infty} x = \infty$$

$$\lim_{x\to\infty} \frac{e^x}{x^{1000000}} \overset{\text{L'Hop}}{=} \lim_{x\to\infty} \frac{e^x}{1000000\, x^{999999}}$$

$$\underline{\underline{\text{L'Hop (a lot)}}} \lim_{x\to\infty} \frac{e^x}{1000000!} = \infty$$

$$\lim_{x\to\infty} \frac{e^x}{x^3} \overset{\text{L'Hop}}{=} \lim_{x\to\infty} \frac{e^x}{3x^2} \overset{\text{L'Hop}}{=} \lim_{x\to\infty} \frac{e^x}{3\cdot 2x} \overset{\text{L'Hop}}{=} \lim_{x\to\infty} \frac{e^x}{3\cdot 2\cdot 1}$$