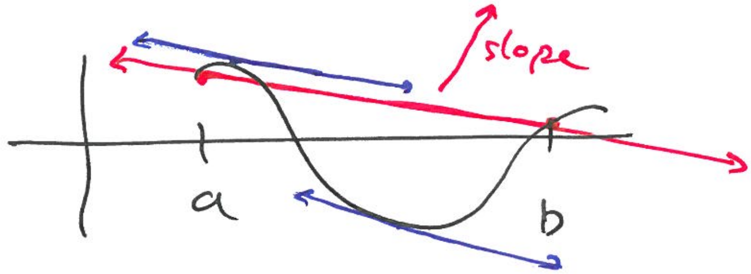
4.3 Mean Value Theorem and Monotonicity

Theorem 4.3.1 (Mean Value Theorem (MVT)). If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there is some $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \rightarrow \text{AROC from a to b}$$

slope



$x \in (a,b)$

$f(x) = \dfrac{f(b)-f(a)}{b-a} > 0$

<u>**Note:**</u> MVT rigorously confirms the following:

$f'(x) > 0$ on (a,b) $\Rightarrow$ f increasing

$f'(x) < 0$ on (a,b) $\Rightarrow$ f decreasing

Example 4.3.1. Find intervals where $f(x) = 10x^2 - 4x + 21$ is increasing or decreasing.

$f'(x) = 20x - 4$

Find CP: $20x - 4 = 0 \Rightarrow x = 1/5$

f increasing: $(1/5, \infty)$

f decreasing: $(-\infty, 1/5)$

$$\xleftarrow{\quad - \quad} \underset{1/5}{|} \xrightarrow{\quad + \quad} f'$$

Theorem 4.3.2 (First Derivative Test). Let $x = c$ be a critical point of $f(x)$, then

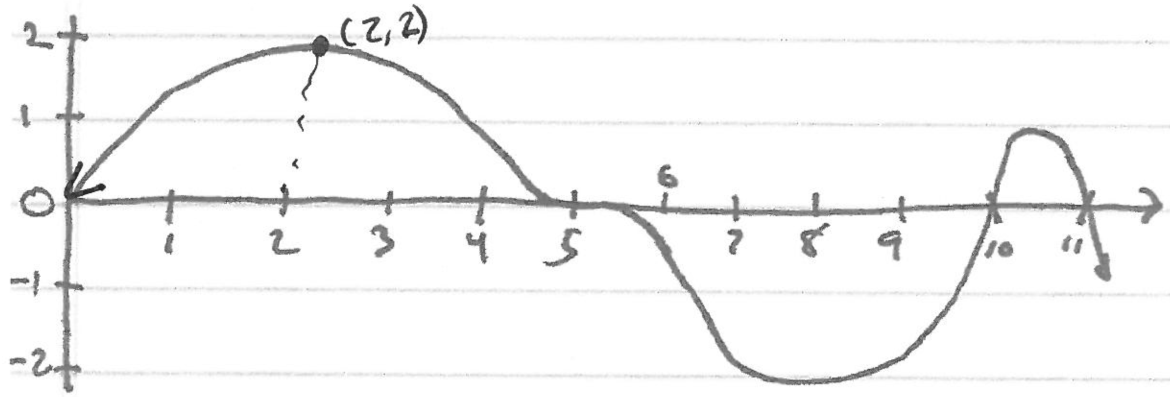
1. if $f'(x)$ changes from negative to positive, then $f(c)$ is a

   local minimum $\cup$

2. if $f'(x)$ changes from positive to negative, then $f(c)$ is a

   local maximum $\cap$

Example 4.3.2. The graph of $f(x)$ is given below.



a. What are the critical points for $f(x)$?

$x = 2, 5, 8, 10.5$ ~~(10 acts like CP)~~

b. What are the local maximums for $f(x)$?

$(2, 2), (10.5, 1)$

$+ \quad 2 \quad - \quad 5 \quad - \quad 8 \quad + \quad 10.5 \quad -$ f'

c. What are the local minimums for $f(x)$?

$(8, -2)$

Example 4.3.3. Let $f(x) = x^4 + x^3$. Find critical points and determine whether $f(x)$ has a local maximum, local minimum, or neither at those points.

$f'(x) = 4x^3 + 3x^2$ (always defined)

$0 = 4x^3 + 3x^2 = x^2(4x+3)$ CP: $x = 0, -3/4$

$- \quad -3/4 \quad + \quad 0 \quad +$ f'

$x=0$ is neither $[(0,0)]$

local min at $(-3/4, f(-3/4))$