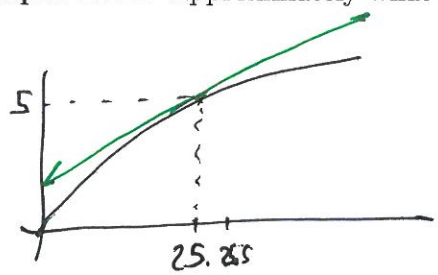
4.1 Linear Approximation

Example 4.1.1. Approximately what is $\sqrt{25.5}$?

tan line at $x=a$

$y = f'(a)(x-a) + f(a)$



$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$$L(x) = y = f'(25)(x-25) + f(25)$$
$$= \frac{1}{10}(x-25) + 5$$

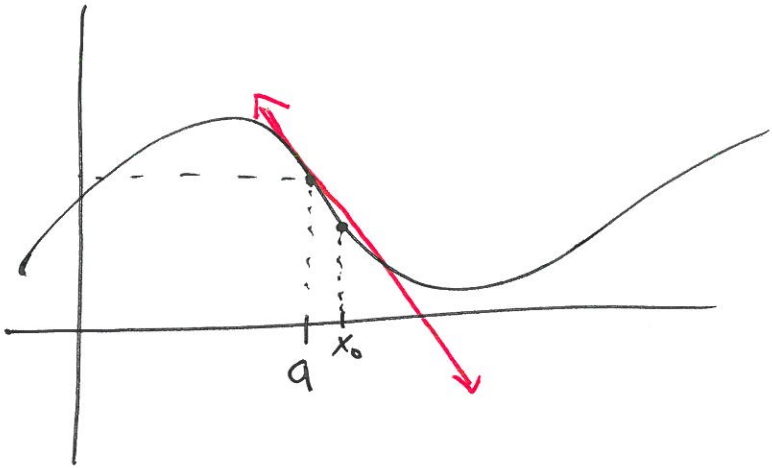
$L(x) =$ "equation of tangent line"

$L(25.5) \approx f(25.5)$

$$L(25.5) = \frac{1}{10}(25.5 - 25) + 5$$
$$= \frac{1}{20} + 5$$
$$= 5.05 \approx \sqrt{25.5}$$

$\sqrt{25.5} = 5.04975\ldots$

Idea of Linear Approximation



know $f(a)$

x_0 close to a

Steps: ① figure out $f(x)$ (and $f(a)$)

② figure out $f'(x)$ (and $f'(a)$)

③ Find tangent line ($y = L(x)$)

④ Find $L(x_0)$

⑤ $f(x_0) \approx L(x_0)$ when x_0 close to a

Also: $\underbrace{f(x_0) - f(a)}_{\Delta f} \approx f'(a)\Delta x$ $\quad (\Delta x = x_0 - a)$

Example 4.1.2. Approximate $\ln(1.07)$ using linearization.

$a = 1,\ x_0 = 1.07$

$f(x) = \ln x \qquad f(1) = \ln 1 = 0$

$f'(x) = \frac{1}{x} \qquad f'(1) = 1$

$L(x) = y = 1(x-1) + 0 = x - 1$

$L(1.07) = 1.07 - 1 = 0.07 \approx \ln(1.07)$

$\ln(1.07) = 0.0676586\ldots$

Example 4.1.3. Approximate $\sqrt[4]{17}$ using linearization.

$f(x) = x^{1/4} \qquad a = 16,\ x_0 = 17\ (\rightarrow \Delta x = 1)$

$f'(x) = \frac{1}{4} x^{-3/4} \qquad f'(16) = \frac{1}{4} \cancel{16}^{-3/4} = \frac{1}{4 \cdot 8} = \frac{1}{32}$

$f(16) = 16^{1/4} = 2$

$L(x) = \frac{1}{32}(x - 16) + 2$

$L(17) = \frac{1}{32} + 2 = \frac{65}{32} = 2.03\ldots$

<u>Check</u>: $\sqrt[4]{17} = 2.0305\ldots$

Example 4.1.4. If $f(17) = 6$ and $f'(17) = -2$, approximate $f(17.2)$.

$$L(x) = f'(17)(x-17) + f(17)$$
$$= -2(x-17) + 6$$

$$f(17.2) \approx L(17.2) = -2(0.2) + 6$$
$$= 5.6$$

Notation 4.1.1.

- $\Delta x = x_0 - a$
- $\Delta f = f(x_0) - f(a)$
- $L(x) = f'(a)(x-a) + f(a)$
- $f(x) \approx L(x)$ for x close to a