

3.5 Higher Derivatives

Example 3.5.1 (from last class). A ball is thrown upward with an initial velocity of $80 \frac{\text{ft}}{\text{sec}}$.

$$s(t) = 80t - 16t^2$$

$$v(t) = 80 - 32t = s'(t)$$

What is $v'(t)$?

$$\begin{aligned} v'(t) &= \text{acceleration} \\ &= -32 \end{aligned}$$

$$f'' = \frac{d^2 f}{dx^2}$$

Definition 3.5.1. The second derivative of $y = f(x)$ is

$$f''(x) = \frac{d}{dx} (f'(x)) = \frac{d^2 f}{dx^2}(x)$$

If $y = f(x)$ is a position function, then

$$y' = \text{velocity}$$

$$y'' = \text{acceleration}$$

$$y''' = \text{jerk}$$

Definition 3.5.2. The n th derivative of $y = f(x)$ is

$$f^{(n)}(x) = \frac{d}{dx} \left(f^{(n-1)}(x) \right) = \frac{d^n f}{dx^n}(x)$$

Example 3.5.2. The position of a particle is given by $f(t) = t^3 - 6t^2 + 9t$. Find the velocity, acceleration, and jerk functions.

$$f'(t) = \text{velocity} = 3t^2 - 12t + 9$$

$$f''(t) = \text{acceleration} = 6t - 12$$

$$f'''(t) = \text{jerk} = 6$$

$$f^{(4)}(t) = \text{snap} = 0$$

$$f^{(5)}(t) = \text{crackle} = 0$$

$$f^{(6)}(t) = \text{pop} = 0$$

Example 3.5.3. $g(x) = xe^x$. What is $g'''(x)$?

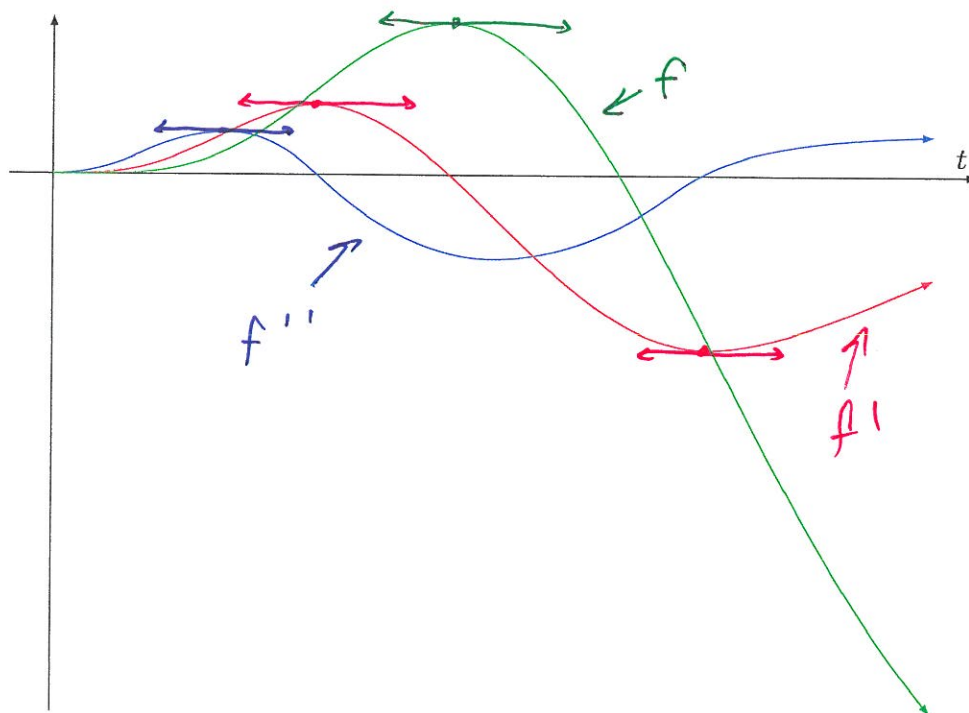
$$g'(x) = (1)e^x + xe^x = e^x(1+x)$$

$$g''(x) = e^x(1+x) + e^x = e^x(2+x)$$

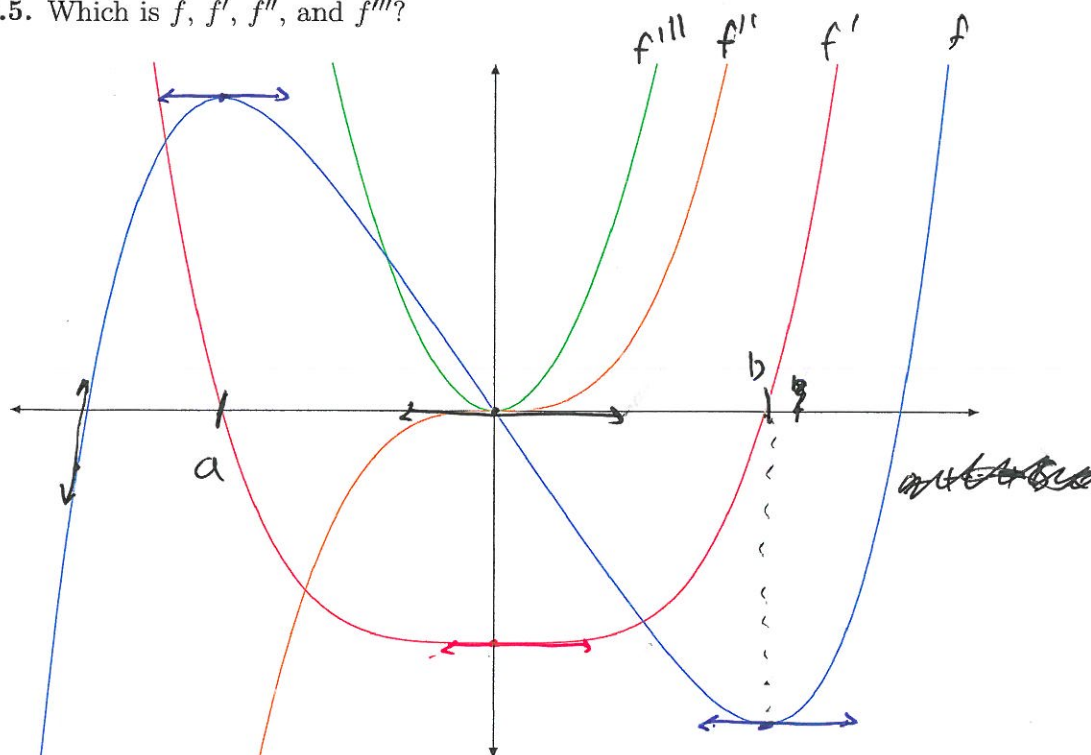
$$g'''(x) = e^x(2+x) + e^x = e^x(3+x)$$

$$g^{(n)}(x) = e^x(n+x).$$

Example 3.5.4. One graph is position, one is velocity, and one is acceleration. Which is which?



Example 3.5.5. Which is f , f' , f'' , and f''' ?



blue's derivative has to be red

red's derivative has to be orange

green's derivative has to be orange (if it is graphed)

