

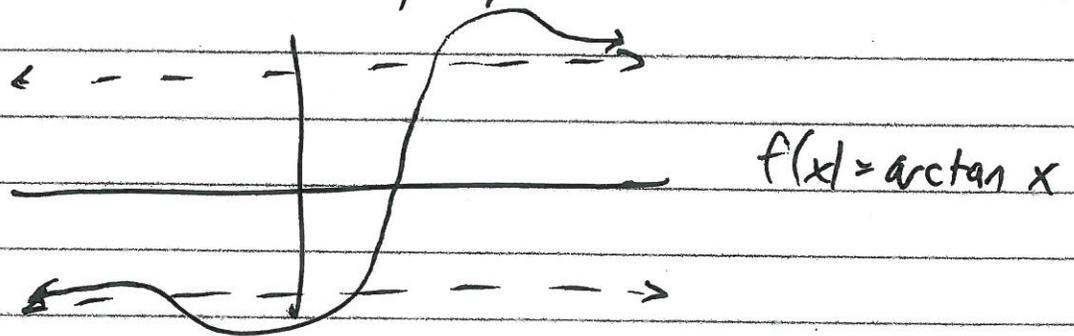
Section 2.7

Definition:

$L = \lim_{x \rightarrow \infty} f(x)$ means $f(x)$ becomes arbitrarily close to L when x increases without bound

$L = \lim_{x \rightarrow -\infty} f(x)$ means $f(x)$ becomes arbitrarily close to L when x decreases without bound

In either case, $y = L$ is called a horizontal asymptote



Tools

$$\lim_{x \rightarrow a} x^n = A$$

$$\lim_{x \rightarrow a} \frac{1}{x^n} = 0$$

$$\lim_{x \rightarrow a} x^n = \begin{cases} A & \text{even} \\ -A & \text{odd} \end{cases}$$

$$\lim_{x \rightarrow \infty} \frac{1}{x^n} = 0$$

$$\lim_{x \rightarrow 3} (x-3) \sin\left(\frac{1}{x-3}\right)$$

$$-1 \leq \sin\left(\frac{1}{x-3}\right) \leq 1$$

$$\begin{array}{ccc} -|x-3| \leq (x-3) \sin\left(\frac{1}{x-3}\right) \leq |x-3| \\ \downarrow & & \downarrow \\ 0 & & 0 \end{array}$$

Plan: M: 2.7, W: 2.8, R: Wash, F: Review M: Test 1

Recall: $\lim_{x \rightarrow \infty} x^n = \infty = \left(\lim_{x \rightarrow \infty} x \right)^n$ $\lim_{x \rightarrow \infty} x^4 = \infty$

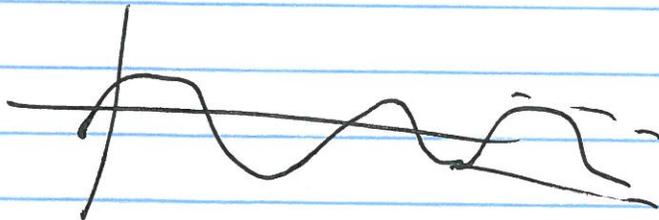
$n > 0$ $\lim_{x \rightarrow \infty} \frac{1}{x^n} = 0$

$n = 1, 2, 3, 4, \dots$ $\lim_{x \rightarrow \infty} x^n = \begin{cases} \infty & n \text{ even} \\ -\infty & n \text{ odd} \end{cases}$

$\lim_{x \rightarrow \infty} \frac{1}{x^n} = 0$

Not every function has a limit at ∞

Ex: $\sin x$



① $\lim_{x \rightarrow \infty} \frac{3x^2 + 20x}{4x^2 + 9} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$
 $= \lim_{x \rightarrow \infty} \frac{3 + \frac{20}{x} \rightarrow 0}{4 + \frac{9}{x^2} \rightarrow 0}$
 $= \frac{3}{4}$

Find the horizontal asymptotes of f

$$f(x) = \frac{\sqrt{36x^4 + 7}}{9x^2 + 4}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{36x^4 + 7}}{9x^2 + 4} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\ = \lim_{x \rightarrow \infty} \frac{\sqrt{36 + 7/x^4}}{9 + 4/x^2} & \quad \frac{1}{x^2} \sqrt{36x^4 + 7} \\ & \quad \left| \left(\frac{1}{x^2} \right)^2 \sqrt{36x^4 + 7} \right. \\ & \quad \left. \frac{1}{x^4} (36x^4 + 7) \right. \\ & \quad \left. \sqrt{36 + 7/x^4} \right. \\ = \frac{\sqrt{36 + 0}}{9 + 0} \\ = \frac{6}{9} \\ = \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{\sqrt{36x^4 + 7}}{9x^2 + 4} &= \lim_{x \rightarrow -\infty} \frac{\sqrt{36 + 7/x^4}}{9 + 4/x^2} \\ &= \frac{\sqrt{36 + 0}}{9 + 0} = \frac{2}{3} \end{aligned}$$

$y = 2/3$ horiz. asy.

$$\lim_{x \rightarrow \infty} (\ln(\sqrt{5x^2+2}) - \ln x)$$

$$= \lim_{x \rightarrow \infty} \left[\ln \left(\frac{\sqrt{5x^2+2}}{x} \right) \right]$$

$$= \ln \left(\lim_{x \rightarrow \infty} \frac{\sqrt{5x^2+2}}{x} \right)$$

$$= \ln(\sqrt{5}) = \frac{1}{2} \ln 5$$

f is cont. at $x=c$

⊗

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} f(\lim_{x \rightarrow c} x) = f(c)$$

~~$\lim_{x \rightarrow c} f(x) = L, f(c) = L$
 $\lim_{x \rightarrow c} g(x)$ exists~~

$$\lim_{x \rightarrow \infty} \frac{\sqrt{5x^2+2}}{x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{5 + \frac{2}{x^2}}}{1} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$$\frac{1}{x} \sqrt{5x^2+2} = \sqrt{\left(\frac{1}{x^2}\right) (5x^2+2)} = \sqrt{\left(\frac{1}{x^2}\right) (5x^2+2)}$$

$$= \sqrt{5 + \frac{2}{x^2}} \quad x^2 \quad \frac{2}{2} \ln x = \frac{1}{2} (2 \ln x) = \frac{1}{2} \ln x^2$$

$$\lim_{x \rightarrow \infty} (\ln(\sqrt{5x^2+2}) - \ln x) = \lim_{x \rightarrow \infty} \left(\frac{1}{2} \ln(5x^2+2) - \frac{2}{2} \ln x \right)$$

$$= \lim_{x \rightarrow \infty} \left(\frac{1}{2} \ln(5x^2+2) - \frac{1}{2} \ln x^2 \right)$$

$$= \frac{1}{2} \lim_{x \rightarrow \infty} (\ln(5x^2+2) - \ln x^2)$$

$$= \frac{1}{2} \lim_{x \rightarrow \infty} \ln \left(\frac{5x^2+2}{x^2} \right)$$