Instructions: Calculators are *not* allowed to be used on this test. There are 100 points. Show all work and simplify your answers unless otherwise specified! Correct answers without work will receive zero points. Also, points will be taken from messy solutions. **Good Luck!** ☺

Question	Points	Score
1	6	
2	5	
3	8	
4	5	
5	6	
6	9	
7	6	
8	6	
9	6	
10	7	
11	8	
12	8	
13	8	
14	12	
Total:	100	

1. (6 points) Suppose the graph of $f(x)$ is given in Figure 1 below.

a. At what x-value(s) does $f'(x)$ not exist?

Solution: $x = 0$

b. Where is $f'(x) > 0$?

Solution: $(-5, -3) \cup (-1, 0)$

c. What is $f'(-3)$?

Solution: $f'(-3) = 0$

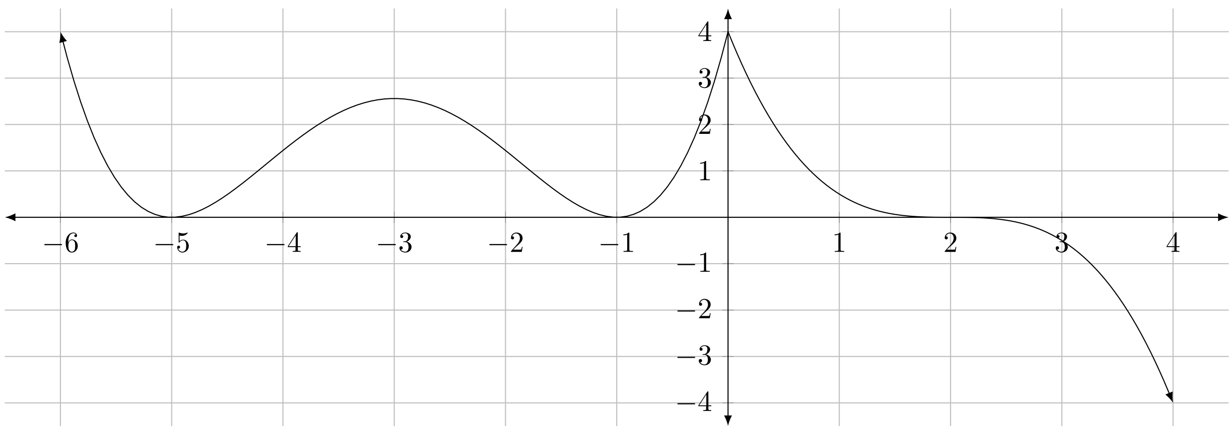


Figure 1: Graph for Questions 1 and 2

2. (5 points) Suppose the graph of $g'(x)$ is given in Figure 1 above.

a. What is $g'(0)$?

Solution: $g'(0) = 4$

b. At what values does g have horizontal tangent lines?

Solution: $x = -5, -1, 2$

c. Where is $g'(x) < 0$?

Solution: $(2, \infty)$

3. (8 points) Use the limit definition of the derivative to compute $f'(x)$ when $f(x) = x^2 + 3x$. (If you do not use the limit definition of the derivative, you will not receive any credit for this problem.)

Solution:

$$\begin{aligned} f'(x) &= \lim_{h \to 0} \frac{(x+h)^2 + 3(x+h) - (x^2+3x)}{h} \\ &= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x}{h} \\ &= \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h} \\ &= \lim_{h \to 0} 2x + h + 3 \\ &= 2x + 3 \end{aligned}$$

4. (5 points) The function $T(x)$ gives the temperature in Knoxville on October 23 at x hours after midnight. Would you expect $T'(8)$ or $T'(16)$ to be larger? Explain your answer.

Solution: The temperatures at 8am and 4pm are both increasing, but at 8am the temperature is increasing faster. So, $T'(8)$ is expected to be larger that $T'(16)$.

5. (6 points) Suppose $f'(4) = 1$, $g(4) = 2$, and $g'(4) = 6$. Do we have enough information to compute $F'(4)$ where $F(x) = f(g(x))$? If so, what is $F'(4)$. If not, what information is missing?

Solution: Since $F'(x) = f'(g(x))g'(x)$, $F'(4) = f'(g(4))g'(4) = f'(2) \cdot 6$. We would need to know $f'(2)$ in order to finish this problem.

Find the derivatives of the following functions. You do **not** have to simplify. **(24 points)**

6. $f(x) = e^{x^2} + e^5 + x\sec x$

Solution: $f'(x) = 2xe^{x^2} + 0 + \sec x + x\sec x\tan x = 2xe^{x^2} + \sec x + x\sec x\tan x$

7. $f(x) = \dfrac{x^2 + 2x - 1}{2x^3 + 1}$

Solution: $f'(x) = \dfrac{(2x+2)(2x^3+1) - (6x^2)(x^2+2x-1)}{(2x^3+1)^2}$

8. $f(x) = (x^2 + 6)^4$

Solution: $f'(x) = 4(x^2+6)^3(2x)$

9. $f(x) = \sec^{-1}(2x)$

Solution: $f'(x) = \dfrac{2}{|2x|\sqrt{(2x)^2 - 1}}$

10. (7 points) Find the equation of the tangent line to $f(x) = \sin(\pi x) + 4x$ at $x = \frac{1}{2}$.

Solution: $f'(x) = \pi\cos(\pi x) + 4$

$f(\frac{1}{2}) = \sin\left(\frac{\pi}{2}\right) + 2 = 3$

$f'(\frac{1}{2}) = \pi\cos(\frac{\pi}{2}) + 4 = 4$

Equation of Tangent Line: $y - 3 = 4(x - \frac{1}{2})$

11. (8 points) Find the second derivative of $f(x) = -\ln(\cos x)$. Make sure to simplify your answer.

Solution: $f'(x) = -\left(\frac{-\sin x}{\cos x}\right) = \tan x$

$f''(x) = \sec^2 x$

12. (8 points) Use logarithmic differentiation to find the derivative of $f(x) = x^{\tan x}$.

Solution:

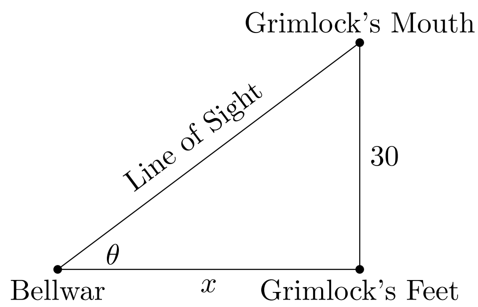
$$\begin{aligned} f'(x) &= x^{\tan x}(\ln x^{\tan x})' \\ &= x^{\tan x}(\tan x \ln x)' \\ &= x^{\tan x}(\sec^2 x \ln x + \frac{1}{x}\tan x) \end{aligned}$$

13. (8 points) Find $\frac{dy}{dx}$ given $xy + \cot x = 7$.

Solution: Taking the derivative: $y + xy' - \csc^2(x) = 0$. So, $xy' = \csc^2(x) - y$ and $y' = \dfrac{\csc^2 x - y}{x}$.

14. (12 points) Bellwar is not doing well. He just realized that Grimlock the tyrannosaurus rex is running straight at him at a rate of 16 ft/s, and Grimlock's mouth is always exactly 30 ft in the air. Bellwar has one last request. Find the rate of change of the angle between the ground and the line of sight into Grimlock's mouth at the exact moment when the distance from Grimlock's feet to Bellwar is 40 ft. You might as well treat Bellwar as lying on the ground, since it just really is not his day.

Solution:



We have that $x' = -16$ and $\tan\theta = \frac{30}{x}$. So,

$$\sec^2(\theta)\theta' = -30x^{-2}x'$$

and

$$\theta' = -30x'x^{-2}\cos^2\theta.$$

When $x = 40$, the length of the hypotenuse in the above triangle is 50. This means $\cos\theta = \frac{40}{50}$ when $x = 40$. Then

$$\theta'|_{x=40} = -30(-16)(40^{-2})\left(\frac{40}{50}\right)^2 = \frac{30 \cdot 16}{50^2} = \frac{24}{125}\frac{\text{rad}}{s}.$$