Instructions: Calculators are allowed to be used on this test. There are 100 points. Show all work and simplify your answers! Correct answers without work will receive zero points. Also, points will be taken from messy solutions. **Good Luck!** ☺

Question	Points	Score
1	5	
2	6	
3	6	
4	5	
5	6	
6	10	
7	4	
8	6	
9	12	
10	6	
11	8	
12	8	
13	8	
14	6	
15	4	
Total:	100	

1. (5 points) Tennessee ran for 148 yards during the football game against Georgia Tech. Does this mean that there was a time when the team had run for 100 yards? Explain your answer using something you learned in this class.

Solution: There are a few possible answers. One option: Yes, yards added is a continuous function and Tenessee started with 0 yards and ended with 148 yards. By the IVT, there was a time when Tennessee had 100 yards. Second option: No, yards added is not continuous.

2. (6 points) Suppose we have that $-1 \leq f(x) \leq 4\sin(x)$. Is this enough to find $\lim_{x\to 0} f(x)$? Explain.

Solution: No, since $-1 < 0 = \lim_{x\to 0} 4\sin(x)$ all we can say is if the limit exists then it is between -1 and 0.

3. (6 points) Assume that

$$\lim_{x\to\infty} f(x) = L \text{ and } \lim_{x\to L} g(x) = \infty.$$

Which of the following statements are correct? (Circle all that apply.)

a. $y = L$ is a horizontal asymptote of f.

b. $y = L$ is a horizontal asymptote of g.

c. $x = L$ is a vertical asymptote of f.

d. $x = L$ is a vertical asymptote of g.

Solution: (a) and (d) are correct.

4. (5 points) Suppose that the function $f(x)$ satisfies

$$\lim_{x\to 0^-} f(x) = 1, \ \lim_{x\to 0^+} f(x) = 1, \text{ and } f(0) = 2.$$

Is this function continuous at $x = 0$? Explain your reasoning.

Solution: No, the function is not continuous at $x = 0$. Since the one-sided limits are both 1, $\lim_{x\to 0} f(x) = 1$. But $f(0) = 2 \neq 1$.

5. (6 points) Where is $f(x) = \ln\left((4x-2)^2\right)$ continuous? Explain your reasoning.

Solution: For $\ln(y)$ to be continuous, $y > 0$. So, we need to find where $(4x-2)^2 > 0$. We get $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$ or $x \neq \frac{1}{2}c$.

6. (10 points) Compute the instantaneous rate of change of $f(x) = \dfrac{1}{-x}$ at $x = 1$ using the limit definition.

Solution:

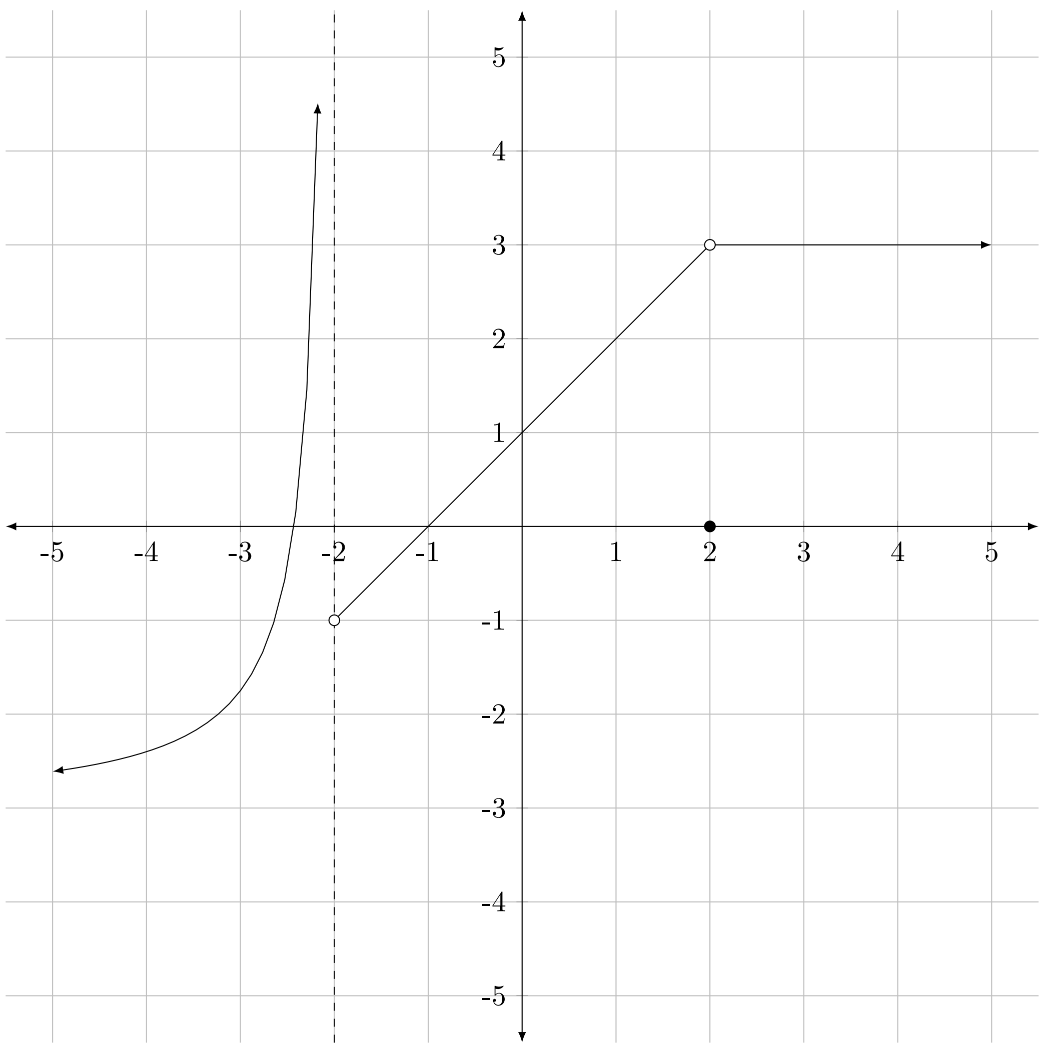
$$\lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{\frac{-1}{x}-(-1)}{x-1} = \lim_{x\to 1} \frac{\frac{-1}{x}+\frac{x}{x}}{x-1} = \lim_{x\to 1} \frac{\frac{x-1}{x}}{x-1} = \lim_{x\to 1} \frac{x-1}{x(x-1)} = \lim_{x\to 1} \frac{1}{x} = 1$$

7. (4 points) Give an example of a function with an indeterminate form at $x = 2$.

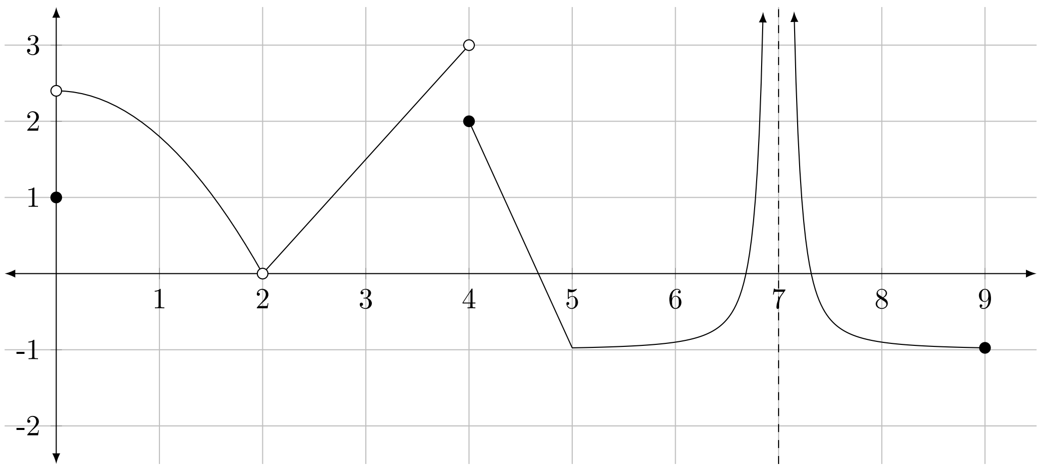
Solution: There are (infinitely) many examples. The simplest are $\frac{x^2-4}{x-2}$ or $\frac{x-2}{x^2-4}$.

8. (6 points) Draw a graph of **one** function $f(x)$ with the following characteristics:

- $f(x)$ is continuous on the intervals $(-\infty, -2)$, $(-2, 2)$, and $(2, \infty)$.
- $\lim_{x\to\infty} f(x) = 3$ and $\lim_{x\to-\infty} f(x) = -3$.
- $\lim_{x\to-2^-} f(x) = \infty$ and $\lim_{x\to-2^+} f(x) = -1$.
- $f(x)$ has a removable discontinuity at 2, but $f(2)$ exists.



9. (12 points) The function $g(x)$ is graphed below. Use this graph to answer the following questions.



a. What is $g(0)$?

Solution: 1

b. What is $\lim_{x\to 2} g(x)$?

Solution: 0

c. What is $\lim_{x\to 4^-} g(x)$?

Solution: 3

d. What is $\lim_{x\to 7} g(x)$?

Solution: ∞

e. What is the average rate of change of $g(x)$ on the interval $[4, 5]$?

Solution: -3=slope of AROC line over [4,5]=$\dfrac{2+1}{-1}$

f. List the x-value(s) in the interval $[0, 9]$ where $g(x)$ is NOT defined.

Solution: x=2,7

Compute the following limits. Show all your work! **(40 points)**

10. $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$

Solution:

$$\begin{aligned}\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right) &= \lim_{x\to 1}\left(\frac{x+1}{(x-1)(x+1)}-\frac{2}{(x-1)(x+1)}\right)\\ &= \lim_{x\to 1}\frac{x-1}{(x-1)(x+1)} = \lim_{x\to 1}\frac{1}{x+1} = \frac{1}{2}\end{aligned}$$

11. $\lim_{x\to -\infty}\frac{\sqrt{100x^2+1}}{5x+1}$

Solution:

$$\lim_{x\to -\infty}\frac{\sqrt{100x^2+1}}{5x+1} = \lim_{x\to -\infty}\frac{\sqrt{100x^2+1}}{5x+1}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to -\infty}\frac{-\sqrt{100+\frac{1}{x^2}}}{5+\frac{1}{x}} = -\frac{10}{5} = -2$$

12. $\lim_{x\to 2}\left(\frac{x^2-4}{x^2+3x-10}\right)$

Solution:

$$\lim_{x\to 2}\left(\frac{x^2-4}{x^2+3x-10}\right) = \lim_{x\to 2}\frac{(x-2)(x+2)}{(x+5)(x-2)} = \lim_{x\to 2}\frac{x+2}{x+5} = \frac{4}{7}$$

13. $\lim_{x \to 0} \frac{\sin(3x)}{\sin(6x)}$

Solution:

$$\lim_{x \to 0} \frac{\sin(3x)}{\sin(6x)} = \lim_{x \to 0} \left(\frac{\sin(3x)}{3x} \frac{6x}{\sin(6x)} \cdot \frac{3}{6} \right) = \frac{1}{2} \lim_{x \to 0} \left(\frac{\sin(3x)}{3x} \frac{6x}{\sin(6x)} \right) = \frac{1}{2}$$

14. $\lim_{x \to 2} \frac{x^2 - 4x + 4}{x + 2}$

Solution:

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x + 2} = \frac{4 - 8 + 4}{4} = 0$$

15. $\lim_{x \to 1^-} \frac{1}{x - 1}$

Solution: Since $x - 1 < 0$ when $x \to 1^-$,

$$\lim_{x \to 1^-} \frac{1}{x - 1} = -\infty$$