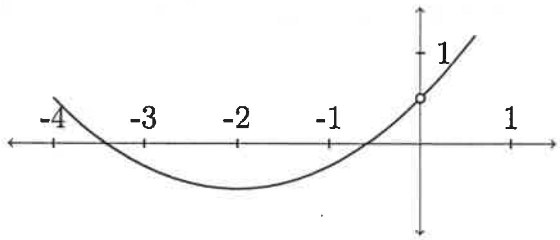
Math 141 Worksheet 2 Name: Key

In this worksheet, you will practice a useful technique for computing limits of certain types of functions at points where the function might not be defined. Remember to write in complete sentences.

1. Now consider the graph of $f(x)$ below:

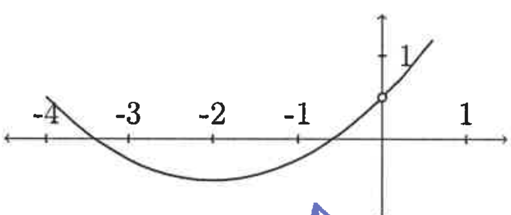


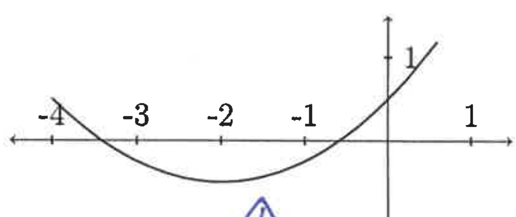
The function $f(x)$ is not defined at $x = 0$, but you can still find $\lim_{x \to 0} f(x)$ by looking at the graph. What is this limit?

$\frac{1}{2}$

It would be nice not to need to look at a graph or to make a table of values to find limits. Fortunately there is a method that often works for computing limits, and we will use it a lot. Let's work through the method now.

2. Below there are two equations and two graphs. Which equation corresponds to which graph?





$$y = \frac{\frac{1}{4}x^3 + x^2 + \frac{1}{2}x}{x}$$

$$y = \frac{1}{4}x^2 + x + \frac{1}{2}$$

Draw lines connecting each equation to its graph. How do you know your answer is correct?

They are the same except at zero. The left hand equation is undefined at 0, but the one on the right is $\frac{1}{2}$ at 0.

Now let $f(x) = (\frac{1}{4}x^3 + x^2 + \frac{1}{2}x)/x$ and let $g(x) = \frac{1}{4}x^2 + x + \frac{1}{2}$. The graphs of $f(x)$ and $g(x)$ above are identical except at $x = 0$; there $g(x)$ is defined and $f(x)$ is not.

3. Why are the graphs of $f(x)$ and $g(x)$ above identical except at $x = 0$? What goes wrong when you try to plug in $x = 0$?

Since $f(x) = \dfrac{x(\frac{1}{4}x^2+x+\frac{1}{2})}{x}$, and $\frac{x}{x} = 1$ except at $x=0$, the graphs are identical except at $x=0$. The function $f(x)$ is $\frac{0}{0}$ at $x=0$.

4. You calculated $\lim_{x \to 0} f(x)$ in Question 3 by looking at its graph. How does this limit compare to the value $g(0)$?

They are equal.

Let's review: we had a function $f(x)$ that had a hole at $x = 0$, and we hoped to find $\lim_{x \to 0} f(x)$ without needing to refer to a graph (or a table). We did this by using a function $g(x)$ that is exactly the same as $f(x)$ except that it is defined at $x = 0$. That is, we filled in the gap in the graph of $f(x)$. Then we could just plug 0 into $g(x)$ to find the limit. This method works because the new function g is *continuous.*

5. Find $\lim_{x \to -1} \dfrac{x^2 - x - 2}{x+1}$. Where is $\dfrac{x^2 - x - 2}{x+1}$ not defined? How did you fix this to find the limit?

$$\frac{x^2-x-2}{x+1} = \frac{(x-2)(x+1)}{x+1} \overset{x \neq -1}{=} x-2$$

not defined at $x=-1$

$$\Rightarrow \lim_{x \to -1} \frac{x^2-x-2}{x+1} = \lim_{x \to -1} x-2 = -3$$

graph