1. Find the instantaneous rate of change of $f(x) = 2x^2 + 10x$ at $x = 3$

$$IROC = \lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = \lim_{x\to 3} \frac{2x^2+10x-48}{x-3}$$

$$= \lim_{x\to 3} \frac{(2x+16)(x-3)}{x-3} = \lim_{x\to 3} 2x+16$$

$$= 22$$

2. Find the instantaneous rate of change of $f(x) = \sqrt{x}$ at $x = 1$

$$IROC = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to 1} \frac{\sqrt{x}-\sqrt{1}}{x-1} = \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$$

$$= \lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}$$

$$= \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{x}+1)}$$

$$= \lim_{x\to 1} \frac{1}{\sqrt{x}+1}$$

$$= \frac{1}{2}$$

OR

$$= \lim_{x\to 1} \frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$= \lim_{x\to 1} \frac{1}{\sqrt{x}+1}$$

$$= \frac{1}{2}$$

3. Find the instantaneous rate of change of $f(x) = x^3 + x$ at $x = 0$

$$IROC = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = \lim_{x\to 0} \frac{x^3+x-0}{x} = \lim_{x\to 0} \frac{x^3+x}{x}$$

$$= \lim_{x\to 0} x^2+1 = 1$$

4. How often is the following statement true:

Whether or not $\lim_{x \to a} f(x)$ exists depends on how $f(a)$ is defined.

(a) sometimes

(b) always

(c) never

Explain your answer:

The limit as $x \to a$ of $f(x)$ could be $f(a)$ or could not be. Since $x \to a$, we don't care what happens to $f(x)$ at $x = a$.

Ex: $\lim_{x \to 0} \frac{x^2}{x} = 0$ even though $f(0)$ is undefined.

5. There are 1261 people in a theater for a play. Does this mean that at some point there were exactly 1000 people in the theater? Explain your thinking.

2 options

(1) Yes, people come in 1 by 1. There were originally 0 people in the theater and then later there were 1261 people. This means at some time there were 1000 people.

(2) No, the number of people in the theater is not continuous, so it could jump from 999 people to 1001 people.