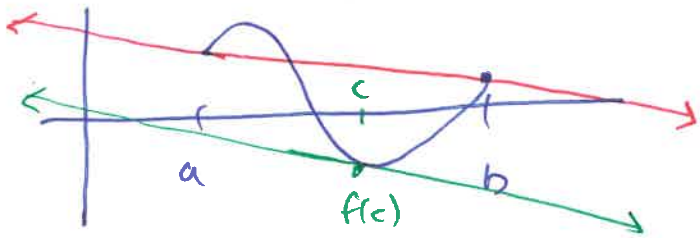
4.3 Mean Value Theorem and Monotonicity

Theorem 4.3.1 (Mean Value Theorem (MVT)). If $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b), then there is some $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \rightarrow \text{AROC}$$



Note: MVT rigorously confirms the following:

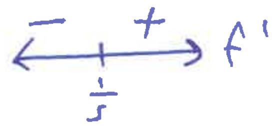
$f'(x) > 0$ on $(a,b) \implies f$ increasing

$f'(x) < 0$ on $(a,b) \implies f$ decreasing

Example 4.3.1. Find intervals where $f(x) = 10x^2 - 4x + 21$ is increasing or decreasing.

$f'(x) = 20x - 4 = 4(5x - 1)$

$f'(x) = 0$ when $x = 1/5$



f is decreasing on $(-\infty, \frac{1}{5})$

f is increasing on $(\frac{1}{5}, \infty)$

Theorem 4.3.2 (First Derivative Test). Let $x = c$ be a critical point of $f(x)$, then

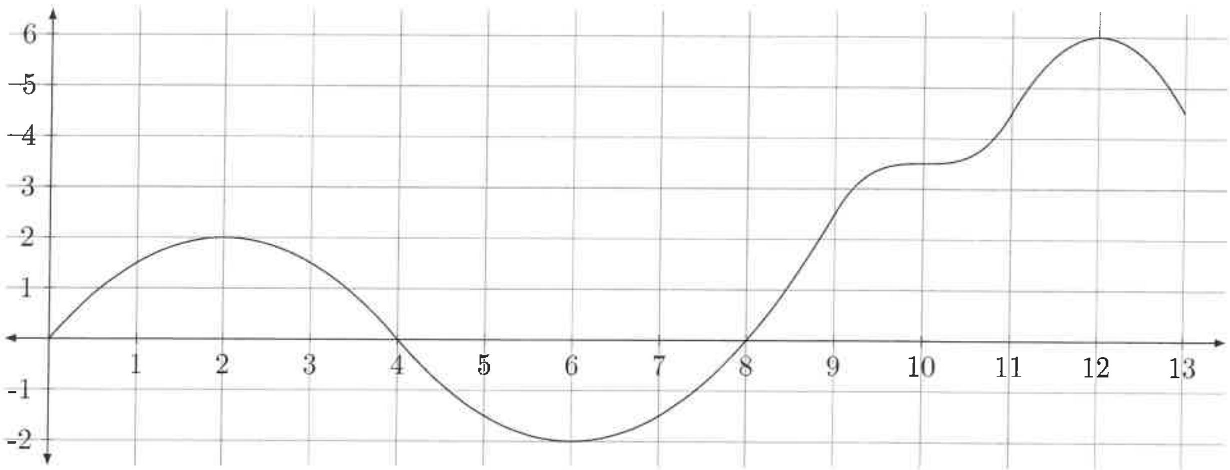
1. if $f'(x)$ changes from negative to positive, then $f(c)$ is a

   local min

2. if $f'(x)$ changes from positive to negative, then $f(c)$ is a

   local max

Example 4.3.2. The graph of $f(x)$ is given below.



a. What are the critical points for $f(x)$?

$x = 2, 6, 10, 12$

b. What are the local maximums for $f(x)$?

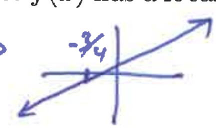
$(2,2), (12,6)$

c. What are the local minimums for $f(x)$?

$(6,-2)$

Example 4.3.3. Let $f(x) = x^4 + x^3$. Find critical points and determine whether $f(x)$ has a local maximum, local minimum, or neither at those points.

$f'(x) = 4x^3 + 3x^2 = x^2(4x+3)$



$f'(x) = 0$ when $x = 0, -3/4 \leftarrow$ CP

$-$ $+$ $+$ f'

$-\frac{3}{4}$ $\quad 0 \leftarrow$ neither

local min: $(-3/4, f(-3/4))$