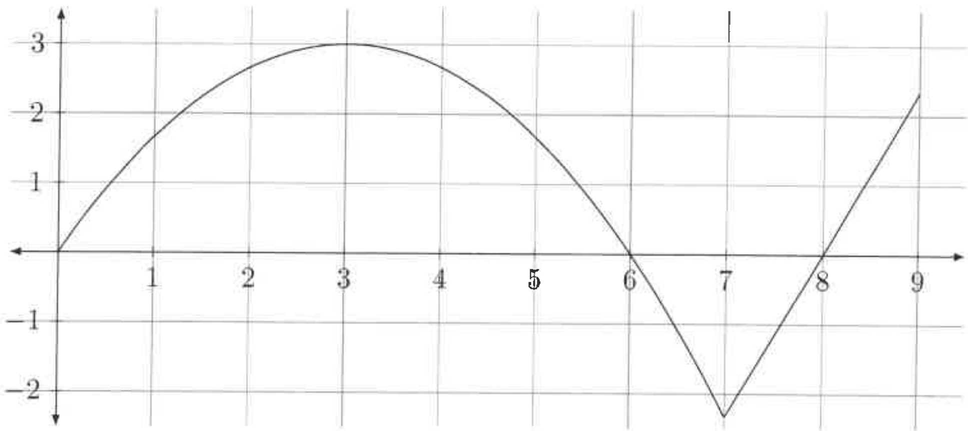
4.2 Extreme Values

Example 4.2.1. The graph of $f(x)$ is given below.



a. What is the maximum of $f(x)$?

3

b. What is the minimum of $f(x)$?

-2.5

c. What happens at these points?

f' DNE or is 0

Definition 4.2.1 (Critical Point). We say $x = c$ is a critical point (x-values) of $f(x)$ when

$f'(c) = 0$ or $f'(c)$ is undefined

Definition 4.2.2 (Extreme Values). The extreme values of $f(x)$ on $[a, b]$ are the minimum and maximum values of $f(x)$ for $x \in [a, b]$.

~~x-values~~ y-values

Definition 4.2.3. $f(c)$ is a local minimum means for input values x that are "close" to c, we have

$f(x) \geq f(c)$

Definition 4.2.4. $f(c)$ is a local maximum means for input values x that are "close" to c, we have

$f(x) \leq f(c)$

$f'(x)=0$ or DNE

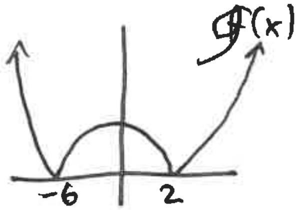
Example 4.2.2. Find the critical points of $f(x) = xe^{2x}$.

$$f'(x) = (2e^{2x})x + e^{2x} = e^{2x} + 2xe^{2x}$$

$$0 = e^{2x} + 2xe^{2x} = e^{2x}(1+2x)$$

$x = -1/2$ is only CP

Example 4.2.3. Find the critical points of $g(x) = |x^2 + 4x - 12|$. $= |(x+6)(x-2)|$



at $x = -6, 2$ $g'(x)$ DNE

on $(-6,2)$ $g(x) = -(x+6)(x-2) = -x^2 - 4x + 12$

$$g'(x) = -2x - 4$$

$$g'(x) = 0 \Rightarrow x = -2$$

CP: $x = -6, -2, 2$

Example 4.2.4. Find the critical points of $h(x) = x + 6x^{\frac{1}{3}}$.

$$h'(x) = 1 + 2x^{-2/3}$$

CP: $x = 0$

$h'(x)$ DNE at $x = 0$

$$0 = 1 + 2x^{-2/3} \Rightarrow 2x^{-2/3} = -1$$

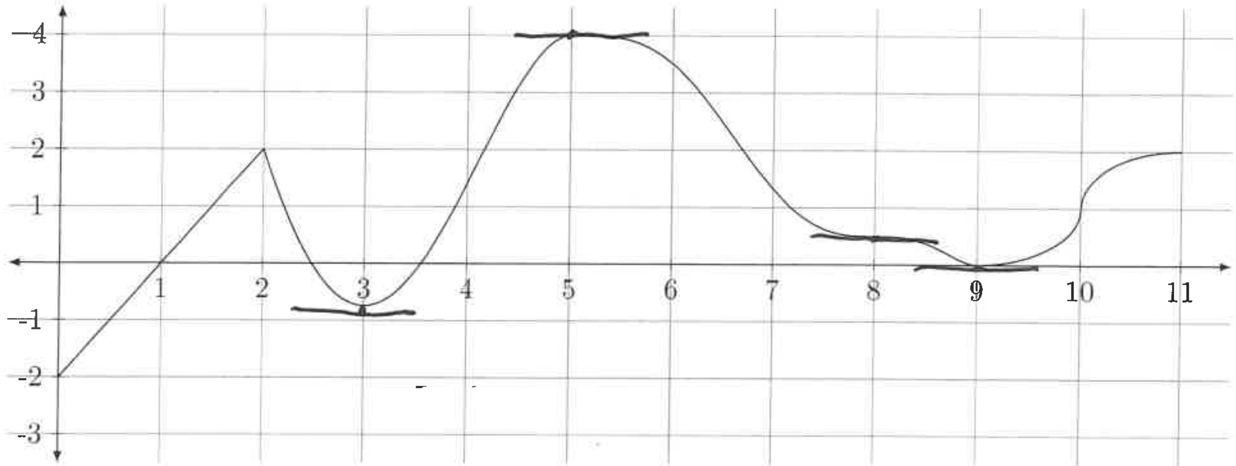
$$\Rightarrow 2 = -x^{2/3}$$

$$\Rightarrow -2 = x^{2/3}$$

$$\Rightarrow -8 = x^2 \rightarrow \text{no real solutions}$$

$$h'(x) \neq 0$$

Example 4.2.5. Use the graph below to answer the following questions.



a. Where is the derivative zero?

x = 3, 5, 8, 9

b. Where is the derivative undefined?

x = 2, 10

c. What are the critical points?

x = 2, 3, 5, 8, 9, 10

d. What is the global maximum?

4

e. What is the global minimum?

-2

What points $x = c$ could give local minimums or local maximums?

$f'(x) = 0$ or DNE $\boxed{\text{or endpoints}}$

What points $x = c$ could give extreme values?

$f'(c) = 0$ or DNE or endpoints

Note: Some CPs may not correspond to local extrema

Theorem 4.2.1 (Extreme Value Theorem (EVT)). A continuous function on a closed interval has a global maximum and global minimum. Further, these occur at

endpoints or CPs

Example 4.2.6. What are the extreme values of $f(x) = xe^{2x}$ on $[-2, 2]$.

$x = -\frac{1}{2}$ is only CP

$f(-2) = -2e^{-4} \approx -0.037$

$f(-\frac{1}{2}) = -\frac{1}{2}e^{-1} \approx -0.184 \leftarrow$ minimum

$f(2) = 2e^{4} \leftarrow$ maximum

Example 4.2.7. What are the extreme values of $g(x) = |x^2 + 4x - 12|$ on $[-5, 3]$.

CP: $x = -6, -2, 2$

$g(-5) = 7$

$g(-2) = 16 \leftarrow$ max

$g(2) = 0 \leftarrow$ min

$g(3) = 9$

Example 4.2.8. What are the extreme values of $k(x) = 4x - \sqrt{x^2+1}$ on $[0, 7]$.

$$k'(x) = 4 - \frac{1}{2}(x^2+1)^{-1/2}(2x)$$

$$= 4 - \frac{x}{\sqrt{x^2+1}}$$

$$\sqrt{x^2+1} = 0$$

$$x^2+1 = 0$$

$$x^2 = -1$$

no real solutions

$$0 = 4 - \frac{x}{\sqrt{x^2+1}}$$

$$\frac{x}{\sqrt{x^2+1}} = 4$$

$$x = 4\sqrt{x^2+1}$$

$$x^2 = 16(x^2+1) = 16x^2 + 16$$

$$15x^2 = -16$$

no real solutions

no CP

$k(0) = -1 \leftarrow$ min

$k(7) = 28 - \sqrt{50} > 0$

$\uparrow$ max

Example 4.2.9. What are the extreme values of $l(x) = \dfrac{1-x}{x^2+3x}$ on $[1,4]$.

$$l(x) = \frac{-(x^2+3x) - (1-x)(2x+3)}{(x^2+3x)^2}$$

$l(x)$ DNE at $x = 0, -3$ (not in $[1,4]$)

$$0 = -(x^2+3x) - (1-x)(2x+3)$$
$$= -x^2 - 3x - (2x + 3 - 2x^2 - 3x)$$
$$= -x^2 - 3x - (-x + 3 - 2x^2)$$
$$= -x^2 - 3x + x - 3 + 2x^2$$
$$= x^2 - 2x - 3$$
$$= (x-3)(x+1)$$

$x = 3, -1$

$l(3) = -1/9 \leftarrow$ min

$l(1) = 0 \leftarrow$ max

$l(4) = -3/28$