3.5 Higher Derivatives

Example 3.5.1 (from last class). A ball is thrown upward with an initial velocity of 80 $\frac{\text{ft}}{\text{sec}}$.

$s(t) = 80t - 16t^2$

$v(t) = -32t + 80$

What is $v'(t)$?

$v'(t) = -32 =$ acceleration $= s''(t)$

$= \frac{d^2}{dx^2} s(t)$

Definition 3.5.1. The second derivative of $y = f(x)$ is

$$f''(x) = \frac{d}{dx}\left(f'(x)\right) = \frac{d^2 f}{dx^2}(x)$$

If $y = f(x)$ is a position function, then

$y' =$ velocity

$y'' =$ acceleration

$y''' =$ jerk

$y'''' =$ snap

$y^{(5)} =$ crackle

$y^{(6)} =$ pop

Definition 3.5.2. The nth derivative of $y = f(x)$ is

$$f^{(n)}(x) = \frac{d}{dx}\left(f^{(n-1)}(x)\right) = \frac{d^n f}{dx^n}(x)$$

Example 3.5.2. The position of a particle is given by $f(t) = t^3 - 6t^2 + 9t$. Find the velocity, acceleration, and jerk functions.

$$\text{velocity} = f'(t) = 3t^2 - 12t + 9$$

$$\text{acceleration} = f''(t) = 6t - 12$$

$$\text{jerk} = f^{(3)}(t) = 6$$

Example 3.5.3. $g(x) = xe^x$. What is $g'''(x)$?

$$g'(x) = (1)e^x + xe^x = e^x(1+x)$$

$$g''(x) = e^x(1+x) + e^x(1) = e^x(2+x)$$

$$g'''(x) = e^x(2+x) + e^x(1) = e^x(3+x)$$

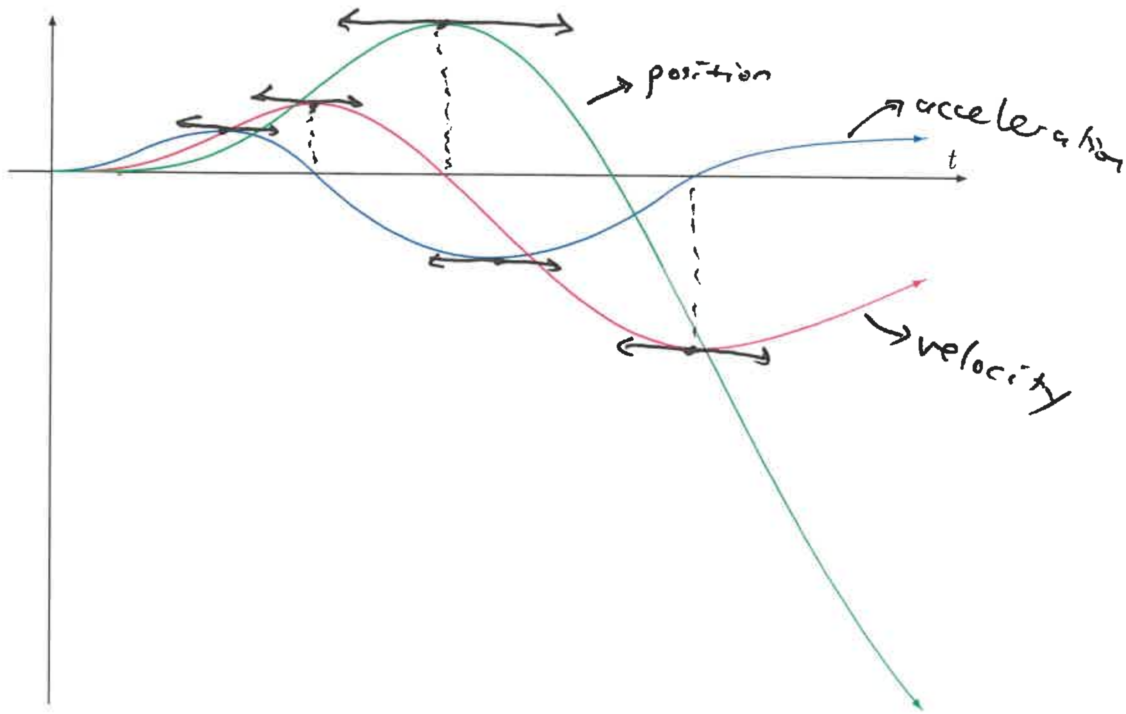
$$\vdots$$

$$g^{(100)}(x) = e^x(100+x)$$

$$\vdots$$

$$g^{(n)}(x) = e^x(n+x)$$

Example 3.5.4. One graph is position, one is velocity, and one is acceleration. Which is which?



Example 3.5.5. Which is f, f', f'', and f'''?