## 2.3 Limit Laws

1. $\lim_{x\to a} k = k$ for any constant $k$
2. $\lim_{x\to a} x = a$

Assume $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist.

3. Sum/Difference Rule: $\lim_{x\to a}(f(x) \pm g(x)) = \left(\lim_{x\to a} f(x)\right) \pm \left(\lim_{x\to a} g(x)\right)$

   Example: $\lim_{x\to 2}(x+10) = \left(\lim_{x\to 2} x\right) + \left(\lim_{x\to 2} 10\right) = 2 + 10 = 12$

4. Constant Multiple Rule: $\lim_{x\to a}(kf(x)) = k \lim_{x\to a} f(x)$ $k$ constant

   Example: $\lim_{x\to 7}(2x-9) = \left(\lim_{x\to 7}(2x)\right) - \left(\lim_{x\to 7} 9\right) = \left(2\lim_{x\to 7} x\right) - 9$

   $= 2\cdot 7 - 9 = 5$

5. Product Rule: $\lim_{x\to a}(f(x)\cdot g(x)) = \left(\lim_{x\to a} f(x)\right)\cdot\left(\lim_{x\to a} g(x)\right)$

   Example: $\lim_{x\to 2}(x^2+3) = \left(\lim_{x\to 2} x^2\right) + \left(\lim_{x\to 2} 3\right) = \left(\lim_{x\to 2} x\right)\left(\lim_{x\to 2} x\right) + 3$

   $= 2\cdot 2 + 3 = 7$

6. Quotient Rule: $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ (Assuming $\lim_{x\to a} g(x) \neq 0$.)

   Example: $\lim_{x\to 1}\frac{x^2-9}{x-3} = \frac{\lim_{x\to 1}(x^2-9)}{\lim_{x\to 1}(x-3)} = \frac{-8}{-2} = 4$

$$\frac{x^2-9}{x-3} = \frac{(x+3)(x-3)}{x-3} \overset{x\neq 3}{=} x+3$$

**7.** Power and Root Rules: For a nonzero integer $n$

- $\star$ $\left(\lim_{x\to a} f(x)\right)^n = \lim_{x\to a} (f(x))^n$
- $\star$ $\sqrt[n]{\lim_{x\to a} f(x)} = \lim_{x\to a} \sqrt[n]{f(x)}$

Example: $\lim_{x\to 4}(x^3 - \sqrt{x} + 1)$

$$= \left(\lim_{x\to 4} x^3\right) - \left(\lim_{x\to 4} \sqrt{x}\right) + \left(\lim_{x\to 4} 1\right)$$

$$= \left(\lim_{x\to 4} x\right)^3 - \sqrt{\lim_{x\to 4} x} + 1$$

$$= 4^3 - \sqrt{4} + 1$$

$$= 64 - 2 + 1$$

$$= 63$$

**Example 2.3.1.** $\lim_{x\to 8}(3x^{\frac{2}{3}} - 16x^{-1})$

$$= \left(\lim_{x\to 8} 3x^{2/3}\right) - \left(\lim_{x\to 8} 16x^{-1}\right)$$

$$= 3\left(\lim_{x\to 8} x^{2/3}\right) - 16\left(\lim_{x\to 8} x^{-1}\right)$$

$$= 3\left(\lim_{x\to 8} x\right)^{2/3} - 16\left(\lim_{x\to 8} x\right)^{-1}$$

$$= 3(8)^{2/3} - 16(8)^{-1}$$

$$= 3\cdot 4 - 16\cdot\frac{1}{8}$$

$$= 12 - 2 = 10$$

**Example 2.3.2.** $\lim_{x\to\frac{1}{2}}(4x^2+1)(6x-1)$ $= \left(\lim_{x\to\frac{1}{2}}(4x^2+1)\right)\left(\lim_{x\to\frac{1}{2}}(6x-1)\right)$

$$\left(\left(\lim_{x\to\frac{1}{2}} 4x^2\right)+\left(\lim_{x\to\frac{1}{2}} 1\right)\right)\left(\left(\lim_{x\to\frac{1}{2}} 6x\right)-\left(\lim_{x\to\frac{1}{2}} 1\right)\right)$$

$$\left(4\left(\tfrac{1}{2}\right)^2+1\right)\left(6\left(\tfrac{1}{2}\right)-1\right)$$

$$\left(\cancel{4}^{1}\left(\tfrac{1}{\cancel{4}}\right)+1\right)\left(3-1\right)$$

$$(1+1)(2)$$

$$(2)(2)$$

$$4$$

**Example 2.3.3.** $\lim_{x\to\frac{11}{2}}(4x^2+6x-5)^{\frac{3}{2}}$

$$= \left(\lim_{x\to\frac{11}{2}}(4x^2+6x-5)\right)^{3/2}$$

$$= \left(4\left(\lim_{x\to\frac{11}{2}} x\right)^2+6\left(\lim_{x\to\frac{11}{2}} x\right)-5\right)^{3/2}$$

$$= \left(4\left(\tfrac{11}{2}\right)^2+6\left(\tfrac{11}{2}\right)-5\right)^{3/2}$$

$$= (121+33-5)^{3/2}$$

$$= (149)^{3/2}$$

**Example 2.3.4.** $\lim_{x\to -1} \frac{(x^2+1)^5}{(x^3+2)(x-3)}$

$$= \frac{\lim_{x\to -1} (x^2+1)^5}{\lim_{x\to -1} \left((x^3+2)(x-3)\right)}$$

$$= \frac{\left(\lim_{x\to -1} (x^2+1)\right)^5}{\left(\lim_{x\to -1} (x^3+2)\right)\left(\lim_{x\to -1} (x-3)\right)}$$

$$= \frac{\left(\lim_{x\to -1} x^2 + \lim_{x\to -1} 1\right)^5}{\left(\lim_{x\to -1} x^3 + \lim_{x\to -1} 2\right)\left(\lim_{x\to -1} x - \lim_{x\to -1} 3\right)}$$

$$= \frac{\left(\left(\lim_{x\to -1} x\right)^2 + 1\right)^5}{\left(\left(\lim_{x\to -1} x\right)^3 + 2\right)(-1-3)}$$

$$= \frac{\left((-1)^2+1\right)^5}{\left((-1)^3+2\right)(-4)}$$

$$= \frac{(1+1)^5}{(-1+2)(-4)}$$

$$= \frac{2^5}{-4} = -8$$