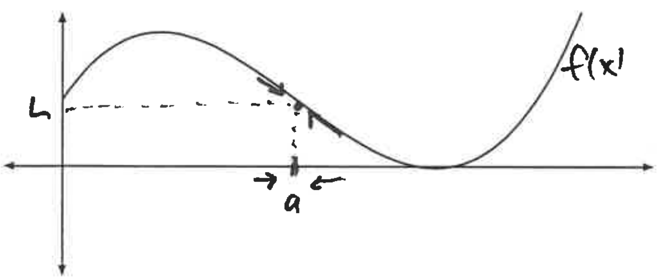
2.2 Numerical and Graphical Limits

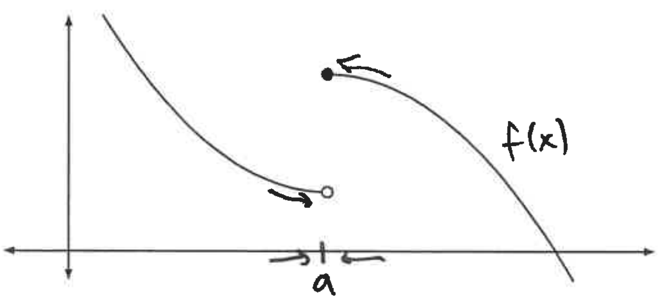
Definition 2.2.1.

$$\lim_{x \to a} f(x) = L$$

means we can make the output values of $f(x)$ arbitrarily close to L by taking x sufficiently close to a (on either side) but not equal to a.
[Read: "the limit of $f(x)$ as x goes to a is L".]



$\lim_{x \to a} f(x) = L$



$\lim_{x \to a} f(x)$ DNE

Does Not exist

Example 2.2.1. Find $\lim_{x \to 1} \underbrace{\frac{x^3 - 1}{x^2 - 1}}_{f(x)} = 1.5$ ← graph says

X	F(x)
0	1
.9	1.42631
.99	1.49251
.999	1.4992

X	F(x)
2	2.33333
1.1	1.5761
1.01	1.5075
1.001	1.5007

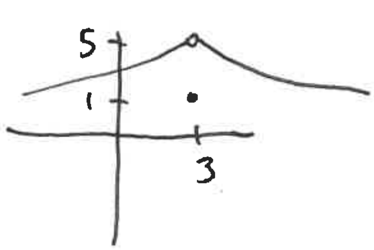
Table says: 1.5

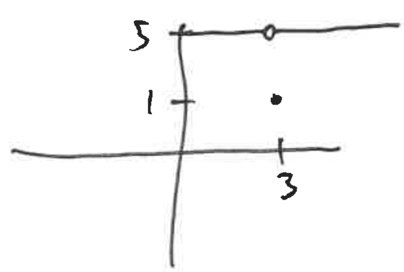
Question: Is it possible for a function $f(x)$ to satisfy:

- $\lim_{x \to 3} f(x) = 5$
- $f(3) = 1$

Why or why not?

Yes!





2.2.1 One-Sided Limits

Definition 2.2.2.

$$\lim_{x \to a^-} f(x) = L$$

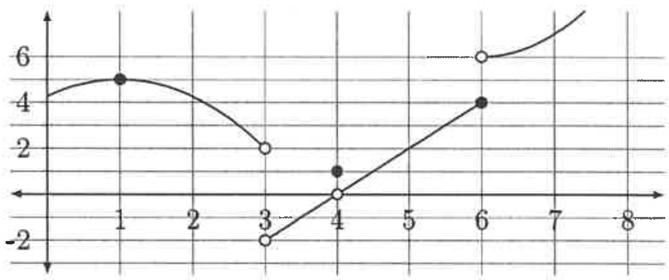
means we can make the output values of $f(x)$ arbitrarily close to L by taking x sufficiently close to a on the left side of a (i.e. $x < a$).

$$\lim_{x \to a^+} f(x) = L$$

means we can make the output values of $f(x)$ arbitrarily close to L by taking x sufficiently close to a on the right side of a (i.e. $x > a$).

$\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^+} f(x) = L$, $\lim_{x \to a^-} f(x) = L$

Example 2.2.2. The graph of $f(x)$ is



Find the following:

$\lim_{x \to 3^-} f(x) =$ 2 $\quad \lim_{x \to 4^-} f(x) =$ 0 $\quad \lim_{x \to 6^-} f(x) =$ 4

$\lim_{x \to 3^+} f(x) =$ -2 $\quad \lim_{x \to 4^+} f(x) =$ 0 $\quad \lim_{x \to 6^+} f(x) =$ 6

$\lim_{x \to 3} f(x) =$ DNE $\quad \lim_{x \to 4} f(x) =$ 0 $\quad \lim_{x \to 6} f(x) =$ DNE

$f(3) =$ undefined $\quad f(4) =$ 1 $\quad f(6) =$ 4

Example 2.2.3. Let

$$f(x) = \begin{cases} x^2 & x < 0 \\ x & 0 \le x \le 1 \\ 1 - x & x > 1 \end{cases}$$

Find the following:

$\lim_{x \to -1^-} f(x) =$ 1 $\quad \lim_{x \to 0^-} f(x) =$ 0 $\quad \lim_{x \to 1^-} f(x) =$ 1 $\quad \lim_{x \to 2^-} f(x) =$ -1

$\lim_{x \to -1^+} f(x) =$ 1 $\quad \lim_{x \to 0^+} f(x) =$ 0 $\quad \lim_{x \to 1^+} f(x) =$ 0 $\quad \lim_{x \to 2^+} f(x) =$ -1

$\lim_{x \to -1} f(x) =$ 1 $\quad \lim_{x \to 0} f(x) =$ 0 $\quad \lim_{x \to 1} f(x) =$ DNE $\quad \lim_{x \to 2} f(x) =$ -1

$f(-1) =$ 1 $\quad f(0) =$ 0 $\quad f(1) =$ 1 $\quad f(2) =$ -1