

Find the equation of the tangent line to the graph of the function  $y = \ln(x^{1/3})$  at the point  $(1, 0)$ .

$$y' = \frac{1}{x^{1/3}} \cdot \frac{1}{3} x^{-2/3} = \frac{1}{3} \cdot \frac{1}{x}$$

$$y'(1) = \frac{1}{3 \cdot 1} = \frac{1}{3}$$

$$y - 0 = \frac{1}{3}(x - 1)$$

Find the derivative of the function  $y = \ln\left(\frac{2-e^x}{2+e^x}\right)$ .

$$y' = \frac{1}{2-e^x} \left( \frac{-e^x(2+e^x) - e^x(2-e^x)}{(2+e^x)^2} \right)$$

Determine an equation of the tangent line to the function  $y = \frac{\ln x}{x}$  at the point  $(1, 0)$ .

$$y' = \frac{\frac{1}{x} \cdot x + x^{-2} \ln x}{x^2}$$

$$y'(1) = \frac{1+0}{1} = 1$$

$$y - 0 = 1(x - 1)$$

Find the derivative of  $y = e^{3x^4-5x}$ .

$$y' = (12x^3 - 5)e^{3x^4 - 5x}$$

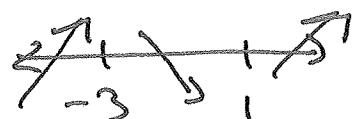
$$\textcircled{1} \quad f(x) = 2x^3 + 6x^2 - 18x - 7$$

$$f'(x) = 6x^2 + 12x - 18 = 6(x^2 + 2x - 3) = 6(x+3)(x-1)$$

$$f''(x) = 12x + 12 = 12(x+1)$$

a) Actually, I'll find intervals of inc/dec.

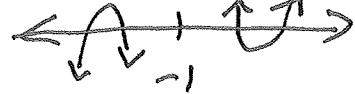
$$0 = f'(x) \Rightarrow x = 1, -3 \quad \text{inc: } (-\infty, -3) \cup (1, \infty)$$



$$\text{dec: } (-3, 1)$$

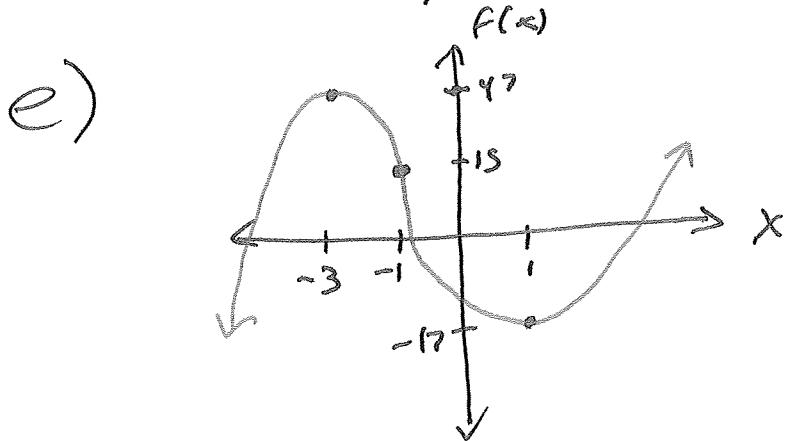
b) max:  $(-3, 47)$  min:  $(1, -15)$

c)  $0 = f''(x) \Rightarrow x = -1$



$$\Rightarrow \begin{array}{l} \text{intervals:} \\ \text{up: } (-1, \infty) \\ \text{down: } (-\infty, -1) \end{array}$$

d) inflection pt:  $(-1, 15)$



$$\textcircled{2} \text{ a) } y' = (8x-3) e^{4x^2-3x}$$

$$b) \frac{1}{(3x^2-5x)^4} \cdot 4(3x^2-5x)^3(6x-5)$$

$$c) y' = 3x^2 \ln x + x^3 \left(\frac{1}{x}\right) - x^2 + 2x e^{x^2}$$

$$d) y' = \frac{\frac{1}{x}(x^3+4) - 3x^2 \ln x}{(x^3+4)^2}$$

$$e) y' = \frac{x^3+4}{x} \cdot \frac{x^3+4 - 3x^2 \cdot x}{(x^3+4)^2}$$

$$f) y' = \frac{1}{x^3-2} 3x^2 + 5e^{x/3} \cdot \frac{1}{3} - 6x$$

$$g) y' = e$$

$$h) y' = 0$$

$$i) y' = \frac{1}{e^x+e^{-x}} \cdot e^x - e^{-x} \quad j) y' = 5e^3 x^4$$

$$\textcircled{3} \quad a) \quad f'(x) = 2e^{2x}$$

slope at  $x=0$ :

$$f'(0) = 2e^0 = 2$$

line:  $\Rightarrow l = 2 \cdot 0 + b$   
 $y = 2x + b \qquad l = b$

eqn of line:

$$y = 2x + l$$

$$b) \quad f'(x) = -2\left(\frac{1}{x}\right) + 5$$

slope at  $x=1$ :

$$f'(1) = -2 \cdot \left(\frac{1}{1}\right) + 5 = 3$$

line:  $\Rightarrow l = 3 \cdot 1 + b$   
 $y = 3x + b \qquad -2 = b$

eqn of line:

$$y = 3x - 2$$

$$⑨ \text{ a) } y' = 2 \cdot e^{-4x} + (-5+2x)e^{-4x} \cdot (-4)$$

$$y'' = 2e^{-4x}(-4) + (-4) \cdot (-5+2x)e^{-4x}(-4)$$

$$+ (-4)(2)e^{-4x}$$

$$= -8e^{-4x} + 16(-5+2x)e^{-4x} - 8e^{-4x}$$

$$\text{b) } y' = 1 - \frac{3}{x}$$

$$y'' = \frac{3}{x^2}$$

$$\textcircled{5} \quad f(x) = x^4 / \ln x$$

$$f'(x) = 4x^3 / \ln x + x^3$$

$$\underline{f'(1) = 1}$$

$$f''(x) = 2x / \ln x + x^2 \left(\frac{1}{x}\right) - 2x$$

$$\begin{aligned} f'(e) &= 2e / (\ln e) + e - 2e \\ &= 2e - e = e \end{aligned}$$

$$f'(x) = \frac{1}{e^x + e^{-x}} (e^x - e^{-x})$$

$$f'(0) = \frac{1}{e^0 + e^0} (e^0 - e^{-0}) = 1 \cdot 0 = 0$$

$$f'(x) = e^{-x/2} \left(-\frac{2x}{2}\right) = -e^{-x/2} \cdot x$$

$$f'(2) = -e^{-4/2} \cdot 2 = -e^{-2} \cdot 2.$$

5. Find the second derivative.

a)  $y = (1+3x)e^{2x}$

$$y' = 2e^{2x}(1+3x) + 3e^{2x}$$

$$y'' = 6e^{2x} + 4e^{2x}(1+3x) + 6e^{2x}$$

b)  $y = 4 - 3 \ln x$

$$y' = -\frac{3}{x}$$

$$y'' = \frac{3}{x^2}$$

6. Find the equation of the tangent line to the function at the given point.

a)  $f(x) = e^x; (0, 1)$

$$f'(x) = e^x$$

$$f'(0) = e^0 = 1$$

line:  $y = x + 1$

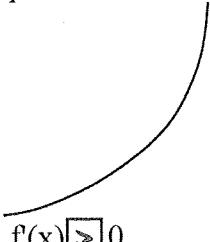
b)  $f(x) = \ln x; (1, 0)$

$$f'(x) = \frac{1}{x}$$

$$f'(1) = 1$$

line:  $y = x - 1$

7. Complete the following inequality with the sign  $<$  or  $>$ .



$$f(x) \boxed{>} 0$$

$$f'(x) \boxed{>} 0$$

8. (a) Find the derivative of the function  $y = \ln\left(\frac{2-e^x}{2+e^x}\right)$ .

(b) Find the derivative of the function  $y = xe^x + \frac{e^x + e^{-x}}{2}$

See other  
worksheet

2. Given the function  $f(x) = x^3 + 3x^2 - 9x + 6$ ,  $f''(x) = 6x + 6 = 6(x+1)$

a) Find the intervals on which  $f$  is concave up or concave down.

$$0 = f''(x) \Rightarrow x = -1$$



b) Find all relative extrema of  $f$ . (State as ordered pairs.)

$$0 = f'(x) \Rightarrow x = 1, -3 \quad \text{max: } (-3, 33)$$



c) Find all inflection points of  $f$ . (State as ordered pairs.)

$$(-1, 17)$$

d) Sketch the graph of  $f$ , showing all extrema. Label at least one value on each axis so I can tell what scale you are using.

